课程大纲
COURSE SYLLABUS

1.	**课程代码/名称** **Course Code/Title**	MAT8027 测度论与积分 Measure Theory and Integration
2.	**课程性质** **Compulsory/Elective**	专业核心课程 Major Core Course
3.	**课程学分/学时** **Course Credit/Hours**	3/48
4.	**授课语言** **Teaching Language**	英文 English
5.	**授课教师 Instructor(s)**	刘博辰副教授 Bochen Liu, Associate Professor
6.	**是否面向本科生开放** **Open to undergraduates or not**	是 yes
7.	**先修要求** **Pre-requisites**	MA301 实变函数， MA202 复变函数， MA302 泛函分析 MA301Theory of Functions of a Real Variable，MA202 Complex Analysis，MA302 Functional Analysis

8. 教学目标 Course Objectives

本课是大学实变函数课程的继续。在大学课程里，学生已经掌握了实轴上的 Lebesgue 测度及积分理论，故而自然地本课将以抽象的测度论开始，再讲抽象可测空间上的积分理论，Lp 空间等。这些内容为其它研究生课程如概率论打下基础。课程的最后部分把学生重新带回 Rn 空间中，学习调和分析的一些最基本的内容， 包括傅氏变换的有界性和傅里叶级数的收敛性等等广泛应用于应用数学和偏微分方程的内容。

This course is the continuation of the undergraduate course “Theory of Functions of a Real Variable”. It starts with the definitions of abstract measures, measurable spaces, measurable functions, etc., then it covers abstract integral theory and Lp spaces. These materials serve as a basis for other related courses, especially Probability Theory. The last part of the course bring the students back to R^n, covering some basic topics in Harmonic Analysis, such as Fourier Transform and the convergence of Fourier series that are useful for Applied Math and PDE.

9. 教学方法 Teaching Methods

将采用传统方式教授此课(版书，课堂讨论，作业，课外答疑，闭卷考试)。强调抽象理论和具体应用的结合。

The course will be taught in the standard way (“chalk and board”, in-class discussion, homework, office hours, closed-book exams). The course is a balanced mix of abstract theories and applications.

10. 教学内容 Course Contents

Section 1	General Measure Spaces: Their Properties and Construction Measures and Measurable Sets Signed Measures: The Hahn and Jordan Decompositions The Carath6odory Measure Induced by an Outer Measure The Construction of Outer Measures The Caratheodory-Hahn Theorem: The Extension of a Premeasure to a Measure
Section 2	Integration Over General Measure Spaces Measurable Functions Integration of Nonnegative Measurable Functions Integration of General Measurable Functions

		The Radon-Nikodym Theorem
	Section 3	The Construction of Particular Measures Product Measures: The Theorems of Fubini and Tonelli Lebesgue Measure on Euclidean Space R" Cumulative Distribution Functions on R and Lebesgue-Stieltjes integral
	Section 4	General L^P Spaces: Completeness, Duality, and Weak Convergence The Completeness of $L^P(X, \mu)$ The Riesz Representation Theorem for the Dual of LP(X, μ), 1 ≤ p < ∞ The Kantorovitch Representation Theorem for the Dual of L∞ (X, μ) Weak Sequential Compactness in LP(X, p.), 1 < p <∞ Weak Sequential Compactness in L1(X, μ): The Dunford-Pettis Theorem
	Section 5	Some Basics in Harmonic Analysis The Fourier transform on L^1 and L^2 Riesz-Thorin interpolation theorem and the Fourier transform on L^p, 1<p<2 The Marcinkiewicz interpolation theorem Hardy-Littlewood maximal function and Hardy-Littlewood maximal inequality Hardy-Littlewood-Sobolev inequality Trigonometric Fourier Series Convergence pointwise: Dirichlet–Dini Theorem, Dirichlet-Jordan Theorem for functions of bounded variation Convergence in L^p, 1<p< ∞
11.	**课程考核 Course Assessment**	
	作业 40%+期中考试（闭卷）20%+期末考试(闭卷) 40% Homework 40%+ Mid-term Exam (closed-book) 20%+Final Exam (closed-book) 40%	
12.	**教材及其它参考资料 Textbook and Supplementary Readings**	
	1. Real Analysis, fourth edition, by Halsey L. Royden and Patrick M. Fitzpatrick, 2010. 2. Topics in Real and Functional Analysis by Gerald Teschl, University of Vienna, 2015 version. 3. Measure and Integral, R. Wheeden and A. Zygmund, 1997.	