课程大纲
COURSE SYLLABUS

1.	课程代码/名称 **Course Code/Title**	MAT8026 高等泛函分析 Advanced Functional Analysis
2.	课程性质 **Compulsory/Elective**	Compulsory
3.	课程学分/学时 **Course Credit/Hours**	3/48
4.	授课语言 **Teaching Language**	English
5.	授课教师 **Instructor(s)**	Raul Ures, Professor;
6.	是否面向本科生开放 **Open to undergraduates or not**	Yes
7.	先修要求 **Pre-requisites**	（如面向本科生开放，请注明区分内容。 If the course is open to undergraduates, please indicate the diff MA301 实变函数 MA202 复变函数 MA302 泛函分析 MA301Theory of Functions of a Real Variable, MA202 Complex Analysis, MA302 Functional Analysis. No differences between undergraduate and graduate students.
8.	教学目标 **Course Objectives**	
	（如面向本科生开放，请注明区分内容。 If the course is open to undergraduates, please indicate the difference.） This course is the continuation of the same-named undergraduate course. It focuses on the classical theory that are important to applications, preparing the students for other related courses and research. No differences between undergraduate and graduate students.	
9.	教学方法 **Teaching Methods**	
	（如面向本科生开放，请注明区分内容。 If the course is open to undergraduates, please indicate the difference.） The course will be taught in the standard way (“chalk and board”, in-class discussion, homework, office hours, closed-book tests). The course is a balanced mix of abstract theories and applications. No differences between undergraduate and graduate students.	
10.	教学内容 **Course Contents** （如面向本科生开放，请注明区分内容。 If the course is open to undergraduates, please indicate the difference.）	
	Section 1	Hahn-Banach Theorem 1.1. The extension theorem 1.2. Hyperplane separation of convex sets 1.3. Applications 1.3.1 Extension of positive linear functionals 1.3.2 Lagrange multipliers of convex programming problems
	Section 2	Weak and weak * convergence 2.1 Weak convergence and weak compactness of unit ball in reflexive

	Banach spaces 2.2 Weak* convergence and weak* sequential compactness—Helly's Theorem 2.3 Banach-Alaoglu Theorem 2.4 Applications 2.4.1 Approximation of the delta-function by continuous functions 2.4.2 Approximate quadrature Existence of PDE via Galerkin's method
Section 3	General spectral theory 3.1. Spectral radius and Gelfand's theorem 3.2. Functional calculus, spectral mapping theorem 3.3. Spectral decomposition/separation theorem 3.4. Isolated eigenvalues 3.4.1. Algebraic multiplicity 3.4.2. Laurent expansion of the resolvent operator near isolated eigenvalue 3.4.3. Stability of a finite set of isolated eigenvalues under small operator perturbation 3.5. Spectrum of the adjoint operator 3.6. The case of unbounded but closed operators
Section 4	Compact operators and Fredholm operators 4.1. Riesz-Schauder theory 4.2. Hilbert-Schmidt theorem, min-max characterization of eigenvalues 4.3. Positive compact operators: Krein-Rutman theorem (for the special case of Banach space C(Q), where Q is a compact Hausdorff space) 4.4. Fredholm operators 4.4.1. Characterization of Fredholm operators, pseudoinverse 4.4.2. Fredholm index: index of product of two operators, constancy of index under small or compact perturbation 4.4.3. Essential spectrum of a bounded operator, and its constancy under compact perturbation 4.5. Applications 4.5.1. Second order elliptic operators 4.5.2. Non-local diffusion operators 4.5.3. Toeplitz operators
Section 5	5. Spectral theory of bounded symmetric, normal and unitary operators 5.1. The spectrum of symmetric operators 5.2. Functional calculus for symmetric operators 5.3. Spectral resolution of symmetric operators 5.4. Absolutely continuous, singular, and point spectra 5.5. The spectral representation of symmetric operators 5.6. Spectral resolution of normal operators 5.7. Spectral resolution of unitary operators 5.8. Examples
Section 6	Unbounded self-adjoint operators 6.1. Spectral resolution via Cayley transform 6.2. The extension of unbounded symmetric operators, deficiency indices 6.3. The Friedrichs extension 6.4. Examples
Section 7	Semigroups of operators 7.1. Strongly continuous one-parameter semigroups

		7.2. The generation of semigroups: Hille-Yosida theorem 7.3. Exponential decay of semigroups 7.4 Examples: semigroups defined by parabolic equations, and by nonlocal diffusion equations
	Section 8	
	Section 9	
	Section 10	
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11.	课程考核 **Course Assessment**	
	（①考核形式 Form of examination；②.分数构成 grading policy；③如面向本科生开放，请注明区分内容。If the course is open to undergraduates, please indicate the difference.） Homework 30%+ Mid-term Exam (closed-book) 30%+Final Exam (closed book) 40%	
12.	教材及其它参考资料 **Textbook and Supplementary Readings**	
	1. Functional Analysis, by Peter Lax. 2. 泛函分析讲义（上、下），张恭庆等编著 Perturbation Theory for Linear Operators, by T. Kato.	