

## 课程大纲 COURSE SYLLABUS

1.	课程代码/名称 <b>Course Code/Title</b>	<b>MAT</b> <b>Topics in Algebra II</b>
2.	课程性质 <b>Compulsory/Elective</b>	选修 elective
3.	课程学分/学时 <b>Course Credit/Hours</b>	3/48
4.	授课语言 <b>Teaching Language</b>	English (英文)
5.	授课教师 <b>Instructor(s)</b>	Prof. Vyacheslav Futorny
6.	是否面向本科生开放 <b>Open to undergraduates or not</b>	是 Yes
7.	先修要求 <b>Pre-requisites</b>	Pre-requisites: Linear Algebra. Previous exposure to Algebra is expected.

### 8. 教学目标 Course Objectives

This graduate-level course will introduce the students to a wide variety of topics in Representation theory, with a particular focus on representations of Lie algebras.

In the beginning of the course, introduction to Representation theory will be given, the students will learn the basic constructions and classical results such as Schur's lemma, Maschke's theorem, Schur–Weyl duality, density theorem, Wedderburn–Artin theorem, representations of quivers and Gabriel's theorem. We then proceed to representations of Lie algebras. The students will learn the following topics: representations of nilpotent and solvable Lie algebras, finite-dimensional representations of semisimple Lie algebras, Weyl character formula, Verma modules theory and category  $\mathcal{O}$ . The students will be exposed to current research topics in Lie theory and related topics.

### 9. 教学方法 Teaching Methods

Lectures by instructor, exercises.

### 10. 教学内容 Course Contents

(如面向本科生开放, 请注明区分内容。 If the course is open to undergraduates, please indicate the difference.)

General outline for some of the topics included in the course is as follows. More precise content depends on the students' background, and time permitting further topics will be included.

<b>Section 1</b>	Introduction (representations and modules, submodules, quotients, tensor product, induced representations, Schur's lemma)
<b>Section 2</b>	Representations of associative algebras: Maschke's theorem, density theorem, Wedderburn–Artin theorem
<b>Section 3</b>	Symmetric groups and Schur–Weyl duality
<b>Section 4</b>	Basics of quiver theory: representations of quivers and Gabriel's theorem
<b>Section 5</b>	Introduction to Lie algebras, representations of nilpotent and solvable Lie algebras
<b>Section 6</b>	Semisimple Lie algebras (root systems, Dynkin diagrams, classification)
<b>Section 7</b>	Weight representations, classification of finite-dimensional representations of semisimple Lie algebras, Weyl theorem

<b>Section 8</b>	Weyl character formula
<b>Section 9</b>	Verma modules and category O (time permitting)

**11. 课程考核  
Course Assessment**

(1考核形式Form of examination; 2.分数构成grading policy; 3如面向本科生开放, 请注明区分内容。 If the course is open to undergraduates, please indicate the difference.)

Exams and participation.

**12. 教材及其它参考资料  
Textbook and Supplementary Readings**

This course will not follow any single textbook for the material. However, here are some suggestions for additional reading.

1. Fulton, W. , Harris, J. Representation Theory. A first course. Graduate Texts in Mathematics. Springer-Verlag, 1991.
2. Humphreys, J. E. Introduction to Lie algebras and representation theory. Springer-Verlag, 1972.
3. P.Etingof et al, Introduction to Representation theory, AMS, Student Mathematical Library, Volume: 59; 2011; 228 pp
4. V.Mazorchuk, Lectures on  $sl_2(\mathbb{C})$ -Modules, World Scientific, 2009

During the course, references to other relevant texts and papers will be provided.