课程大纲
COURSE SYLLABUS

1.	课程代码/名称 **Course Code/Title**	MAT7093 随机分析 Stochastic Analysis
2.	课程性质 **Compulsory/Elective**	选修 Elective
3.	课程学分/学时 **Course Credit/Hours**	3/48
4.	授课语言 **Teaching Language**	中英双语 Chinese-English bilingual
5.	授课教师 **Instructor(s)**	李立颖，副教授 Liying Li, Associate Professor
6.	是否面向本科生开放 **Open to undergraduates or not**	是 Yes
7.	先修要求 **Pre-requisites**	MA215 概率论，MA208 应用随机过程，MA201a 或 MA201b 常微分方程，MA411 测度论与积分，MA302 泛函分析 Probability Theory, Applied Stochastic Processes, Ordinary Differential Equation, Measure Theory and Integration, Functional Analysis.

8. 教学目标
Course Objectives

在概率论和随机过程论基础上，掌握随机分析的基础理论与方法，为进一步研究随机控制、金融数学、金融工程等学科提供必要的随机分析基础。

The main objectives of this course are, based on the preliminary knowledge of probability theory and stochastic processes, to master the basic theory and methods in stochastic analysis and to provide necessary foundations and background in further learning on stochastic control, financial mathematics and financial engineering.

9. 教学方法
Teaching Methods

PPT 结合板书授课。

Teach with PPT and blackboards.

10. 教学内容
Course Contents
（如面向本科生开放，请注明区分内容。 If the course is open to undergraduates, please indicate the difference.）

Section 1 (3 hours)	**预备知识**：随机过程，高斯空间与高斯过程，无穷维测度空间。 **Preliminary**: stochastic processes, Gaussian spaces and Gaussian processes, measure theory on infinite-dimensional spaces.
Section 2 (9 hours)	**布朗运动与连续鞅**：布朗运动构造，布朗运动的路径性质；停时，连续时间鞅，停时定理；Doob-Meyer 分解定理；连续平方可积鞅。 **Brownian motion and continuous martingales:** construction of

		Brownian motions, path properties; stopping times, continuous-time martingales, Optional Sampling Theorem; the Doob-Meyer decomposition, continuous, square-integrable martingales; Brownian Motion.
	Section 3 (12 hours)	**随机积分：**伊藤积分的构造；局部化方法；变量替换公式（伊藤公式）；鞅表示定理； Girsanov 定理。 **Stochastic integrals:** Construction of the Itô integral; technique of localization; the change-of-variable formula (Itô's Formula); representations of continuous martingales in terms of Brownian motion; The Girsanov theorem.
	Section 4 (6 hours)	**布朗运动与偏微分方程：**Feller 半群，生成元；布朗运动与调和函数；布朗运动与热方程，Feynman--Kac 公式。 **Brownian motion and partial differential equations**: Feller semi-groups, generators; Brownian motion and harmonic functions; Brownian motion and the heat equation, Feynman--Kac Formula.
	Section 5 (12 hours)	**随机微分方程**：强解，存在唯一性；弱解，鞅问题；线性方程；一般的 Feynman--Kac 公式 **Stochastic differential equations:** strong solutions, existence and uniqueness; weak solutions, the martingale problem; linear equations; general Feynman--Kac Formula.
	Section 6 (6 hours)	**随机分析应用选讲：**局部时，推广的伊藤公式；线性倒向随机微分方程，适应解的存在唯一性。 **Selected topics on applications:** local time, generalized Itô's Formula; linear backward stochastic differential equations, existence and uniqueness of adapted solutions.

11. 课程考核
Course Assessment

20%课堂表现 + 30%作业 + 50%期末考试

20% class participation + 30% homework assignments + 50% final exam.

12. 教材及其它参考资料
Textbook and Supplementary Readings

1. Le Gall, Jean-François. Brownian Motion, Martingales, and Stochastic Calculus. Vol. 274. Graduate Texts in Mathematics. Cham: Springer International Publishing, 2016.
2. I. Karatzas and S.E. Shreve. Brownian Motion and Stochastic Calculus, 2nd ed., Springer-Verlag, New York, 1998.
3. J. Yong and X. Y. Zhou. Stochastic Controls: Hamiltonian Systems and HJB Equations, Springer-Verlag, New York, 1999.
4. D. Revuz and M. Yor. Continuous Martingales and Brownian Motion, 3rd ed., Springer-Verlag, New York, 1999.
5. N. Ikeda and S. Watanabe. Stochastic Differential Equations and Diffusion Processes, North-Holland Publishing Company, New York, 1981.