

## 偏微分方程 (H) 课程大纲

- 1、2020 秋季学期——2021 春季学期
- 2、2021 秋季学期起

## 课程详述

### COURSE SPECIFICATION

以下课程信息可能根据实际授课需要或在课程检讨之后产生变动。如对课程有任何疑问，请联系授课教师。

The course information as follows may be subject to change, either during the session because of unforeseen circumstances, or following review of the course at the end of the session. Queries about the course should be directed to the course instructor.

1.	<b>课程名称 Course Title</b>	偏微分方程(H) Partial Differential Equations(H)
2.	<b>授课院系 Originating Department</b>	数学系 Mathematics
3.	<b>课程编号 Course Code</b>	MA336
4.	<b>课程学分 Credit Value</b>	3
5.	<b>课程类别 Course Type</b>	专业核心课 Major Core Courses
6.	<b>授课学期 Semester</b>	2020 年秋季 Fall
7.	<b>授课语言 Teaching Language</b>	英语 English
8.	<b>授课教师、所属学系、联系方式 (如属团队授课, 请列明其他授课教师) Instructor(s), Affiliation &amp; Contact (For team teaching, please list all instructors)</b>	崔书墨 (cuism@sustech.edu.cn), 数学系 Shumo Cui (cuism@sustech.edu.cn), Department of Mathematics
9.	<b>实验员/助教、所属学系、联系方式 Tutor/TA(s), Contact</b>	待公布 To be announced
10.	<b>选课人数限额(可不填) Maximum Enrolment (Optional)</b>	

11. 授课方式 Delivery Method	讲授 Lectures	习题/辅导/讨论 Tutorials	实验/实习 Lab/Practical	其它(请具体注明) Other (Please specify)	总学时 Total
	48	0	0	0	48
学时数 Credit Hours					
12. 先修课程、其它学习要求 Pre-requisites or Other Academic Requirements	常微分方程 A (MA201a) 或者常微分方程 A (H) Ordinary Differential Equations A (MA201a) or Ordinary Differential Equations A (H) (MA230)				
13. 后续课程、其它学习规划 Courses for which this course is a pre-requisite	本课程为大学理工科以及金融数学专业的基础课程。 This course is for undergraduates who are science, engineering, mathematics, or financial mathematics majors.				
14. 其它要求修读本课程的学系 Cross-listing Dept.	无 None				

### 教学大纲及教学日历 SYLLABUS

#### 15. 教学目标 Course Objectives

偏微分方程理论研究一个方程是否有满足某些条件的解（解的存在性），有多少个解（解的惟一性），解的各种性质以及求解方法等等，并且还要尽可能地用偏微分方程来解释和预见自然现象以及把它用之于各门科学和工程技术。偏微分方程理论的形成和发展都与物理学和其他自然科学的发展密切相关，并彼此促进和推动。本课程介绍三种主要的偏微分方程类型：扩散，椭圆和双曲。本课程将带领学生学习偏微分方程的基本概念，理论和方法，重点是对 PDE 模型及其在其他学科中的应用的理解。

Partial differential equation theory studies whether an equation has a solution that satisfies certain conditions (the existence of a solution), how many solutions (the uniqueness), the various properties of the solution, the solution method, etc. Partial differential equations are used to explain and foresee natural phenomena and to apply them to various scientific and engineering disciplines. The formation and development of partial differential equation theory are closely related to the development of physics and other natural sciences. This course introduces three types of partial differential equations: diffusion, ellipse, and hyperbolic. This course will lead students to learn the basic concepts, theories and methods of partial differential equations, with an emphasis on understanding the PDE model and its application in other disciplines.

#### 16. 预达学习成果 Learning Outcomes

通过本课程，学生将掌握偏微分方程的基本概念，理论和方法，掌握运输方程，热方程，拉普拉斯方程，泊松方程和波动方程的物理背景和数学推导。掌握特征方法，变量分离方法，能量方法，基本解方法，格林函数方法和 d'Alembert 公式。掌握最大最小原则及其应用，并对线性方程和 nonlinear 方程的区别有所了解。

Through this course, students will master the basic concepts, theories and methods of partial differential equations, master the physical background and mathematical derivation of transportation equation, heat equation, Laplace equation, Poisson equation and wave equation. Master the characteristic method, variable separation method, energy method, basic solution method, Green's function method and d'Alembert formula. Master the principle of maximum and minimum and its application. Understand the differences between linear and nonlinear equations.

#### 17. 课程内容及教学日历（如授课语言以英文为主，则课程内容介绍可以用英文；如团队教学或模块教学，教学日历须注明主讲人）

**Course Contents (in Parts/Chapters/Sections/Weeks. Please notify name of instructor for course section(s), if this is a team teaching or module course.)**

课时数	章节	Content	内容
2	1	Definition of Partial Differential Equations (PDEs); order, linearity, homogeneity, superposition principle; general solutions, initial and boundary conditions; classifications of second order PDEs; examples of PDEs	基本概念: 偏微分方程, 阶, 线性, 齐次性, 叠加原理, 通解, 初边值问题, 二阶偏微分方程的分类, 偏微分方程举例
2	2.1	Transport equation: derivation	运输方程及其推导
2	2.2	First-order linear PDEs: method of characteristics, general solutions and break-down of smoothness	一阶线性偏微分方程: 特征线法, 通解, 爆破
1	3.1	Heat equation and reaction-diffusion equation: derivation	热方程和反应扩散方程及其推导
1	3.2	Boundary conditions for heat and diffusion equations	边界条件
1	3.3	Uniqueness of solution of heat equation via energy method	解的唯一性 (能量方法)
2	3.4	Method of separation of variables	分离变量法
3	3.5	Eigenvalue problems: Sturm-Liouville theory and eigen-expansion	特征值问题: Sturm-Liouville 理论
3	3.6	Non-homogeneous problem	非齐次问题
3	3.7	Fundamental solution of heat equation	热方程的基本解
2	3.8	The Maximum principles	极值原理
1	3.9*	Fourier Transform	傅里叶变换
1	3.10*	Orthogonality and Fourier Series	正交性和傅里叶级数
1	4.1	Laplace and Poisson equations	Laplace 方程和 Poisson 方程
3	4.2	Separation of variables	分离变量法
3	4.3	Fundamental solution of Laplace equation	Laplace 方程的基本解
3	4.4	Green's identities and applications	Green 公式及其应用
1	4.5	Maximum-minimum principle	极值原理
3	4.6	Method of Green's function	Green 函数法
2	5.1	Wave equation: string vibration	波方程: 弦振动
1	5.2	Energy and uniqueness	能量和唯一性
2	5.3	Method of separation of variables	分离变量法
3	5.4	d'Alembert formula and wave propagation	d'Alembert 公式和波传播

1	6.1*	First Order and Second Order Traffic Flow Models	一阶和二阶交通流模型
1	6.2*	Shallow Water Equations and Steady States	浅水方程及其稳态

18. 教材及其它参考资料 Textbook and Supplementary Readings

<p><b>Textbook:</b></p> <p>Lecture notes on Partial Differential Equations, Tao Tang and Xuefeng Wang.</p> <p><b>Reference:</b></p> <ol style="list-style-type: none"> <li>1. A First Course in Partial Differential Equations with Complex Variables and Transform Methods, H. F. Weinberger, 1995.</li> <li>2. Partial Differential Equations: An Introduction, second edition, Walter A. Strauss, 2008.</li> <li>3. Applied Partial Differential Equations, fifth edition, Richard Haberman, 2013.</li> </ol>
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课程评估 ASSESSMENT

19. 评估形式 Type of Assessment	评估时间 Time	占考试总成绩百分比 % of final score	违纪处罚 Penalty	备注 Notes
出勤 Attendance				
课堂表现 Class Performance				
小测验 Quiz				
课程项目 Projects				
平时作业 Assignments	3 hours per week	20		
期中考试 Mid-Term Test	2 hours	40		
期末考试 Final Exam	3 hours	40		
期末报告 Final Presentation				
其它（可根据需要 改写以上评估方 式） Others (The above may be modified as necessary)				



20. 记分方式 **GRADING SYSTEM**

- A. 十三级等级制 **Letter Grading**  
 B. 二级记分制（通过/不通过） **Pass/Fail Grading**

课程审批 **REVIEW AND APPROVAL**

21. 本课程设置已经过以下责任人/委员会审议通过  
**This Course has been approved by the following person or committee of authority**



## 课程详述

### COURSE SPECIFICATION

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1.	课程名称 <b>Course Title</b>	偏微分方程(H) <b>Partial Differential Equations(H)</b>
2.	授课院系 <b>Originating Department</b>	数学系 Mathematics
3.	课程编号 <b>Course Code</b>	MA336
4.	课程学分 <b>Credit Value</b>	3
5.	课程类别 <b>Course Type</b>	专业核心课 Major Core Courses
6.	授课学期 <b>Semester</b>	2021 年秋季 Fall
7.	授课语言 <b>Teaching Language</b>	英语 English
8.	授课教师、所属学系、联系方式（如属团队授课，请列明其他授课教师） <b>Instructor(s), Affiliation &amp; Contact</b> (For team teaching, please list all instructors)	王晓明教授， 数学系 Prof. XIAOMING WANG, Mathematics, wangxm@sustech.edu.cn  Time and Place: Monday 4:00--5:50 PM, Room 101, Building 1, Lychee Garden Wednesday (even weeks) 4:00 - 5:50 PM, Room 102 Building 2, Lychee Garden
9.	实验员/助教、所属学系、联系方式 <b>Tutor/TA(s), Contact</b>	曹祎宁博士 caoyin@sustech.edu.cn Dr. Yining Cao caoyin@sustech.edu.cn
10.	选课人数限额(可不填) <b>Maximum Enrolment (Optional)</b>	30

11. 授课方式 Delivery Method	讲授 Lectures	习题/辅导/讨论 Tutorials	实验/实习 Lab/Practical	其它(请具体注明) Other (Please specify)	总学时 Total
	48	0	0	0	48
学时数 Credit Hours					
12. 先修课程、其它学习要求 Pre-requisites or Other Academic Requirements	常微分方程 A (MA201a) 或者常微分方程 A (H) Ordinary Differential Equations A (MA201a) or Ordinary Differential Equations A (H) (MA230)				
13. 后续课程、其它学习规划 Courses for which this course is a pre-requisite	本课程为大学理工科以及金融数学专业的基础课程。 This course is for undergraduates who are science, engineering, mathematics, or financial mathematics majors.				
14. 其它要求修读本课程的学系 Cross-listing Dept.	无 None				

### 教学大纲及教学日历 SYLLABUS

#### 15. 教学目标 Course Objectives

偏微分方程理论研究一个方程是否有满足某些条件的解（解的存在性），有多少个解（解的惟一性），解的各种性质以及求解方法等等，并且还要尽可能地用偏微分方程来解释和预见自然现象以及把它用之于各门科学和工程技术。偏微分方程理论的形成和发展都与物理学和其他自然科学的发展密切相关，并彼此促进和推动。本课程介绍三种主要的偏微分方程类型：扩散，椭圆和双曲。本课程将带领学生学习偏微分方程的基本概念，理论和方法，重点是对 PDE 模型及其在其他学科中的应用的理解。

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Through this course, students will master the basic concepts, theories and methods of partial differential equations, master the physical background and mathematical derivation of transportation equation, heat equation, Laplace equation, Poisson equation and wave equation. Master the characteristic method, variable separation method, energy method, the method of fundamental solutions, Green's function method and d'Alembert formula. Master the principle of maximum and minimum and its application. Understand the differences between linear and nonlinear equations.

#### 17. 课程内容及教学日历（如授课语言以英文为主，则课程内容介绍可以用英文；如团队教学或模块教学，教学日历须注明主讲人）

**Course Contents (in Parts/Chapters/Sections/Weeks. Please notify name of instructor for course section(s), if this is a team teaching or module course.)**



1. 热传导方程 (4h)
2. 分离变量法(4h)
3. 傅立叶级数 (8h)
4. 波动方程：弦振动和薄膜振动 (4h)
5. Sturm-Liouville 特征值问题(6h)
6. 高维偏微分方程 (6h)
7. 非齐次问题 (2h)
8. 格林函数 (8h)
9. 运输方程及特征线方法 (2h)

1. Heat equation (Ch.1, 4h)
2. Separation of variables (Ch2, 4h)
3. Fourier series (Ch. 3, 8h)
4. Wave equation: vibration of strings and membranes (Ch. 4, 4h)
5. Sturm-Liouville eigenvalue problems (Ch. 5, 6h)
6. Higher-dimensional PDEs (Ch. 7, 6h)
7. Non-homogeneous problems (Ch.8, 2h)
8. Green's function (Ch. 9, 11, 8h)
9. Transport equation and the method of characteristics (Ch. 12, 2h)

Notice that the time allocated is an estimate only. The instructor will take an interactive approach in teaching which may lead to a slightly different schedule.

**18. 教材及其它参考资料 Textbook and Supplementary Readings**

**Textbook:**

Applied Partial Differential Equations, fifth edition, Richard Haberman, 2013.

**Reference:**

1. A First Course in Partial Differential Equations with Complex Variables and Transform Methods, H. F. Weinberger, 1995.
2. Partial Differential Equations: An Introduction, second edition, Walter A. Strauss, 2008.
3. 数学物理方程 第三版 谷超豪李大潜陈恕行 高等教育出版社， 2012

**课程评估 ASSESSMENT**

19. 评估形式 Type of Assessment	评估时间 Time	占考试总成绩百分比 % of final score	违纪处罚 Penalty	备注 Notes
出勤 Attendance		5		
课堂表现 Class Performance		5		
小测验 Quiz				
课程项目 Projects				
平时作业 Assignments	3-5 hours per week	30		

期中考试 Mid-Term Test	2 hours	30		
期末考试 Final Exam	3 hours	30		
期末报告 Final Presentation				
其它（可根据需要 改写以上评估方式） Others (The above may be modified as necessary)				

20. 记分方式 GRADING SYSTEM

- A. 十三级等级制 Letter Grading  
 B. 二级记分制（通过/不通过） Pass/Fail Grading

课程审批 REVIEW AND APPROVAL

21. 本课程设置已经过以下责任人/委员会审议通过  
This Course has been approved by the following person or committee of authority

