

课程详述

COURSE SPECIFICATION

以下课程信息可能根据实际授课需要或在课程检讨之后产生变动。如对课程有任何疑问，请联系授课教师。

The course information as follows may be subject to change, either during the session because of unforeseen circumstances, or following review of the course at the end of the session. Queries about the course should be directed to the course instructor.

1.	课程名称 Course Title	线性代数精讲 Advanced Linear Algebra
2.	授课院系 Originating Department	数学系 Department of Mathematics
3.	课程编号 Course Code	MA109
4.	课程学分 Credit Value	4 学分 4 Credits
5.	课程类别 Course Type	通识选修课程 General Education (GE) Elective Courses
6.	授课学期 Semester	春季 Spring / 秋季 Fall
7.	授课语言 Teaching Language	英文 English / 中英双语 English & Chinese
8.	授课教师、所属学系、联系方式 (如属团队授课, 请列明其他授课教师) Instructor(s), Affiliation & Contact (For team teaching, please list all instructors)	李才恒, 教授, 数学系 慧园 3 栋 528 邮箱: lich@sustc.edu.cn 电话: 0755-88018755 Caiheng Li, Professor, Department of Mathematics Room 528, Block 3, Wisdom Garden. email: lich@sustc.edu.cn phone: 0755-88018755 陈懿茂 数学系 慧园 3 栋 508 huy@sustech.edu.cn Chen Yi mao Department of Mathematics Block 3, Room508, Wisdom Valley Chenym@sustech.edu.cn
9.	实验员/助教、所属学系、联系方式 Tutor/TA(s), Contact	待公布 To be announced

10. Maximum Enrolment (Optional)	选课人数限额(可不填)				
11. Delivery Method	讲授 Lectures	习题/辅导/讨论 Tutorials	实验/实习 Lab/Practical	其它(请具体注明) Other (Please specify)	总学时 Total
学时数 Credit Hours	64	32	N/A		96
12. Pre-requisites or Other Academic Requirements	先修课程、其它学习要求 线性代数 A 或者 线性代数 B Linear Algebra A or Linear Algebra B				
13. Courses for which this course is a pre-requisite	后续课程、其它学习规划 本课程是许多其它数学课程所需要。 Needed for many other mathematics courses.				
14. Cross-listing Dept.	其它要求修读本课程的学系 数学系, 物理, 计算机科学等. Physics, Computer Science, etc.				

教学大纲及教学日历 SYLLABUS

15. 教学目标 Course Objectives

本课程是为数学系本科生所设计的必修课, 也为其它专业需要更多代数知识的学生所设置的, 比如物理学和计算机系的学生。旨在使学生理解线性代数的更多更深的基本知识, 并掌握严格的代数学证明方法。

This course is designed for students of mathematics department, and also for students who need more algebra, such as students in department of physics and department of computer science. It enables students to understand more fundamental contents of linear algebra, and to use algebraic methods to solve problems.

16. 预达学习成果 Learning Outcomes

通过对本课程的学习, 学生将:

1. 理解和掌握以下概念: 向量空间、线性映射以及在不同空间上定义的线性算子。
2. 熟练掌握线性映射的特征值和特征向量以及不变子空间。
3. 熟练掌握内积空间的重要性质及其应用。
4. 熟练掌握定义在内积空间, 复向量空间, 实向量空间上的算子的相关理论及其应用。
5. 从算子的角度理解迹和行列式的概念。

After completing this course, students should be able to:

1. Understand the definitions of vector spaces, linear maps and linear operators defined on various vector spaces.
2. Understand the concepts of eigenvectors, eigenvalues of linear maps and invariant subspaces of vector spaces.
3. Have a deep understanding of properties of inner product spaces and their applications.
4. Understand the theory and applications of operators on inner product spaces, operators on complex vector spaces, and operators on real vector spaces.
5. Understand the concepts of trace and determinant from the point of view of operators.

17. 课程内容及教学日历（如授课语言以英文为主，则课程内容介绍可以用英文；如团队教学或模块教学，教学日历须注明主讲人）

Course Contents (in Parts/Chapters/Sections/Weeks. Please notify name of instructor for course section(s), if this is a team teaching or module course.)

Chapter 1 Vector Spaces (4 hours)

1.A \mathbb{R}^n and \mathbb{C}^n : Complex Numbers, Lists, \mathbb{F}^n , Digression on Fields

1.B Definition of Vector Space

1.C Subspaces: Sums of Subspaces, Direct Sums

第一章 向量空间（4 小时）

1.A \mathbb{R}^n 与 \mathbb{C}^n

1.B 向量空间的定义

1.C 子空间

Chapter 2 Finite-Dimensional Vector Spaces (4 hours)

2.A Span and Linear Independence: Linear Combinations and Span, Linear Independence.

2.B Bases

2.C Dimension

第二章 有限维向量空间 (4 小时)

2.A 张成空间与线性无关

2.B 基

2.C 维数

Chapter 3 Linear Maps (10 hours)

3.A The Vector Space of Linear Maps: Definitions and Examples of Linear Maps, Algebraic Operations on $L(V,W)$.

3.B Null Spaces and Ranges: Null Space and Injectivity, Range and Surjectivity, Fundamental Theorem of Linear Maps.

3.C Matrices: Representing a Linear Map by a Matrix, Addition and Scalar Multiplication of Matrices, Matrix Multiplication.

3.D Invertibility and Isomorphic Vector Spaces: Invertible Linear Maps, Isomorphic Vector Spaces, Linear Maps Thought of as Matrix Multiplication, Operators.

3.E Products and Quotients of Vector Spaces: Products of Vector Spaces, Products and Direct Sums, Quotients of Vector Spaces.

3.F Duality; The Dual Space and the Dual Map, The Null Space and Range of the Dual of a Linear Map, The Matrix of the Dual of a Linear Map, The Rank of a Matrix.

第三章 线性映射（10 小时）

3.A 向量空间的线性映射

3.B 零空间与值域

3.C 矩阵

3.D 可逆性与同构的向量空间

3.E 向量空间的积与商

3.F 对偶

Chapter 4 Polynomials (4 hours)

Complex Conjugate and Absolute Value, Uniqueness of Coefficients for Polynomials, The Division Algorithm for Polynomials, Zeros of Polynomials, Factorization of Polynomials over \mathbb{C} , Factorization of Polynomials over \mathbb{R} .

第四章 多项式（4 小时）

Chapter 5 Eigenvalues, Eigenvectors, and Invariant Subspaces (6 hours)

5.A Invariant Subspaces: Eigenvalues and Eigenvectors, Restriction and Quotient Operators.

5.B Eigenvectors and Upper-Triangular Matrices: Polynomials Applied to Operators, Existence of Eigenvalues, Upper-Triangular Matrices.

5.C Eigenspaces and Diagonal Matrices

第五章 本征值、本征向量、不变子空间 (6 小时)

5.A 不变子空间

5.B 本征向量与上三角矩阵

5.C 本征空间与对角矩阵

Chapter 6 Inner Product Spaces (6 hours)

6.A Inner Products and Norms: Inner Products, Norms.

6.B Orthonormal Bases: Linear Functionals on Inner Product Spaces.

6.C Orthogonal Complements and Minimization Problems: Orthogonal Complements, Minimization Problems.

第六章 内积空间 (6 小时)

6.A 内积与范数

6.B 规范正交基

6.C 正交补与极小化问题

Chapter 7 Operators on Inner Product Spaces (8 hours)

7.A Self-Adjoint and Normal Operators: Adjoints, Self-Adjoint Operators, Normal Operators.

7.B The Spectral Theorem: The Complex Spectral Theorem, The Real Spectral Theorem.

7.C Positive Operators and Isometries: Positive Operators, Isometries.

7.D Polar Decomposition and Singular Value Decomposition: Polar Decomposition, Singular Value Decomposition.

第七章 内积空间上的算子 (8 小时)

7.A 自伴算子与正规算子

7.B 谱定理

7.C 正算子与等距同构

7.D 极分解与奇异值分解

Chapter 8 Operators on Complex Vector Spaces (8 hours)

8.A Generalized Eigenvectors and Nilpotent Operators: Null Spaces of Powers of an Operator, Generalized Eigenvectors, Nilpotent Operators.

8.B Decomposition of an Operator: Description of Operators on Complex Vector Spaces, Multiplicity of an Eigenvalue, Block Diagonal Matrices, Square Roots.

8.C Characteristic and Minimal Polynomials: The Cayley-Hamilton Theorem, The Minimal Polynomial.

8.D The Jordan Form.

第八章 复向量空间上的算子 (8 小时)

8.A 广义本征向量和幂零算子

8.B 算子的分解

8.C 特征多项式和极小多项式

8.D 若尔当形

Chapter 9 Operators on Real Vector Spaces (4 hours)

9.A Complexification: Complexification of a Vector Space, Complexification of an Operator, The Minimal Polynomial of the Complexification, Eigenvalues of the Complexification, Characteristic polynomial of the Complexification.

9.B Operators on Real Inner Product Spaces

Normal Operators on Real Inner Product Spaces, Isometries on Real Inner Product Spaces.

第九章 实向量空间上的算子 (4 小时)

9.A 复化

9.B 实内积空间上的算子

Chapter 10 Trace and Determinant (4 hours)

10.A Trace: Change of Basis: Trace: A Connection Between Operators and Matrices.

10.B Determinant: Determinant of an Operator, Determinant of a Matrix, The Sign of the Determinant, Volume.

第十章 迹与行列式 (4 小时)

10.A 迹

10.B 行列式

18. 教材及其它参考资料 Textbook and Supplementary Readings

教材 (Textbook): Sheldon Axler, **Linear Algebra Done Right**, Third Edition, (UTM), Springer Cham Heidelberg New York Dordrecht London, ISSN 0172-6056.

参考书 (Supplementary Readings): 姚慕生, 吴泉水, 高等代数学, 第二版, 复旦大学出版社, ISBN: 9787309059632.

课程评估 ASSESSMENT

19. 评估形式 Type of Assessment	评估时间 Time	占考试总成绩百分比 % of final score	违纪处罚 Penalty	备注 Notes
出勤 Attendance		5		
课堂表现 Class Performance				
小测验 Quiz		15		
课程项目 Projects				
平时作业 Assignments		10		
期中考试 Mid-Term Test		30		
期末考试 Final Exam		40		
期末报告 Final Presentation				
其它 (可根据需要 改写以上评估方 式) Others (The above may be modified as necessary)				



20. 记分方式 **GRADING SYSTEM**

- A. 十三级等级制 **Letter Grading**
 B. 二级记分制（通过/不通过） **Pass/Fail Grading**

课程审批 **REVIEW AND APPROVAL**

21. 本课程设置已经过以下责任人/委员会审议通过
This Course has been approved by the following person or committee of authority

