课程大纲
COURSE SYLLABUS

1.	课程代码/名称 **Course Code/Title**	**MAT8029 应用数学方法** **MAT8029 Methods of Applied Math**
2.	课程性质 **Compulsory/Elective**	必修 **Compulsory**
3.	课程学分/学时 **Course Credit/Hours**	3 学分/48 学时
4.	授课语言 **Teaching Language**	英文 English
5.	授课教师 **Instructor(s)**	张振
6.	是否面向本科生开放 **Open to undergraduates or not**	是
7.	先修要求 **Pre-requisites**	MA201a 常微分方程 a，MA303 偏微分方程 Ordinary and Partial Differential Equations

8. 教学目标
Course Objectives

掌握计算和应用数学模型建立和分析的基本方法

Master the basic methods of modelling and analysis in computational and applied mathematics

9. 教学方法
Teaching Methods

专题性质授课，并辅以前沿课题应用

Teaching in topics, and application to cutting edge problems

10. 教学内容
Course Contents
（如面向本科生开放，请注明区分内容。 If the course is open to undergraduates, please indicate the difference.）

Section 1	Perturbation methods for algebraic equations: i) Regular perturbation ii) Singular perturbation iii) Non-integer powers iv) Logarithms v) Eigenvalue problems
Section 2	Local analysis for the solutions to ODEs i) Series solutions to ODEs ii) Types of points of homogeneous linear ODEs iii) Frobenius methods

	iv) Method of dominant balance
Section 3	Asymptotic expansions i) Asymptotic sequences ii) Asymptotic power series iii) Properties of asymptotic series iv) Asymptotic series vs. convergent series v) Other asymptotic expansions
Section 4	Asymptotic expansion of integrals i) Direct expansion of integrands ii) Integration by parts iii) Laplace's method iv) Method of stationary phase v) Method of steepest descent
Section 5	Introduction to global analysis and perturbation methods i) Example of perturbation method ii) Asymptotic expansion in \epsilon iii) An example with direct expansion iv) Regular vs. singular perturbation problems
Section 6	Boundary layer theory i) Boundary layer problems v) Boundary layer theory vi) Location of boundary layer vii) Higher order boundary layer theory viii) Boundary layer thickness and distinguished limit ix) Boundary layer problem including logarithm term x) Multiple boundary layers xi) Internal boundary layer xii) Boundary layer in PDE problem
Section 7	WKB theory i) Introduction ii) WKB theory iii) More remarks on the asymptotic expansions iv) Problems with turning points v) Eigenvalue problems (Storm-Liville problem) vi) Application to wave equations vii) Inhomogeneous linear equations
Section 8	Multiple scale analysis i) Secular terms ii) Method of strained coordinates iii) Multiple scale analysis iv) Slowly varying coefficients v) Method of averaging
Section 9	Homogenization method i) Background ii) 1D problem iii) Multi-dimensional problems iv) Porous medium flow – Darcy's law
Section 10	Bifurcation and stability i) Linearized stability of steady states

		ii) Limit cycle and Hopf bifurcation iii) System of ODEs
	Section 11	Basic calculus of variation i) Introductory example – geodesic on a sphere ii) First variation: Euler-Lagrange equation iii) Isoperimetric problems – Catenary problem iv) Holonomic constraints: Lagrange multipliers v) Free boundary problems – natural boundary condition vi) Hamilton-Jacobi system vii) Noether's theorem* viii) Second variation*
	Section 12	From calculus of variation to optimal control theory: an introduction* i) Basic formulation ii) Pontryagin's maximum principle iii) Dynamical programming and Hamilton-Jacobi-Bellman equation iv) Linear quadratic regulator

11. 课程考核
Course Assessment

平时作业（35%）+出勤（5%）+闭卷期中考试（20%）+闭卷期末考试（40%）

assignments (35%), attendance (5%), closed-book midterm exam (20%) and final exam (40%)

12. 教材及其它参考资料
Textbook and Supplementary Readings

1*. (Textbook) C. M. Bender and S. A. Orszag, Advanced mathematical methods for scientists and engineers, Springer, 1999.

2*. M. H. Holmes, Introduction to perturbation methods, Springer-Verlag, 1995.

3*. A. W. Bush, Perturbation methods for engineers and scientists, Boca Raton, 1992.

4*. E. J. Hinch, Perburbation methods, Cambridge University Press, 1991.

5^. A. Bensoussan, J.-L. Lions, G. Papanicolaou, Asymptotic analysis for periodic structures, North-Holland, Oxford, 1978.

6^. U. Hornung, Homogenization and porous media, Springer, 1997.

7^. G. A. Pavliotis, A. M. Stuart, Multiscale methods: averaging and homogenization, Springer, 2008.

8#. Bruce van Brunt, The Calculus of Variations, Springer-Verlag, 2004.

9#. 张恭庆，变分学讲义，高等教育出版社，2011.

10#. M. Giaquinta and S. Hildebrandt, Calculus of Variations, Vol. I and II, Springer, 1996.

11#. Daniel Liberzon, Calculus of Variations and Optimal Control Theory, Princeton

University Press, 2012.

The books with * are standard textbook about perturbation methods; books with ^ concerns mathematical homogenization methods; books with # discuss calculus of variations.