课程详述

COURSE SPECIFICATION

　　以下课程信息可能根据实际授课需要或在课程检讨之后产生变动。如对课程有任何疑问，请联系授课教师。

　　The course information as follows may be subject to change, either during the session because of unforeseen circumstances, or following review of the course at the end of the session. Queries about the course should be directed to the course instructor.

1.	课程名称 Course Title	现代概率论 Advanced Probability
2.	授课院系 Originating Department	数学系 Department of Mathematics
3.	课程编号 Course Code	MAT8011
4.	课程学分 Credit Value	3
5.	课程类别 Course Type	专业选修课 Major Elective Courses
6.	授课学期 Semester	秋季 Fall
7.	授课语言 Teaching Language	英文 English
8.	授课教师、所属学系、联系方式（如属团队授课，请列明其他授课教师） Instructor(s), Affiliation& Contact （For team teaching, please list all instructors）	熊捷，讲座教授，数学系 慧园 3 栋 527 Jie Xiong, Chair Professor, Department of Mathematics Block 3 Room.527, Wisdom Valley
9.	实验员/助教、所属学系、联系方式 Tutor/TA(s), Contact	待公布 To be announced
10.	选课人数限额(可不填) Maximum Enrolment （Optional）	

11.	授课方式 Delivery Method	讲授 Lectures	习题/辅导/讨论 Tutorials	实验/实习 Lab/Practical	其它(请具体注明) Other（Please specify）	总学时 Total
	学时数 Credit Hours	48				48

	先修课程、其它学习要求 **12.** **Pre-requisites or Other Academic Requirements**	MA215 Probability Theory， MA301 Theory of Functions of a Real Variable；MA215 概率论，MA301 实变函数
13.	后续课程、其它学习规划 **Courses for which this course is a pre-requisite**	
14.	其它要求修读本课程的学系 **Cross-listing Dept.**	

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教学大纲及教学日历 SYLLABUS

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15.　　教学目标 **Course Objectives**

Modern probability theory is the basis of Stochastic Processes Theory, Stochastic Analysis and Financial Mathematics. This course is designed to introduce the basic concepts and methods in modern probability theory, which will lay a solid foundation for further study. We will discuss several important kinds of convergences and their theory, conditional expectation and martingale method. After the study of this course, students should not only deeply understand and master concepts and theorems in modern probability theory, but also have the ability to use them in many different problems.

现代概率论是随机过程论，随即分析和金融数学的基础。本课程旨在介绍现代概率论的基本概念与方法，为今后的学习打下坚实的基础。本课程重点介绍了几种重要的收敛概念和理论，条件期望与鞅方法。通过本课程的学习，学生不仅可以深刻理解与掌握各种概念与定理，还应具有把他们运用到不同问题中的能力。

16.　　预达学习成果 **Learning Outcomes**

After learning this course, students should be able

1.to deeply understand and master the basic concepts and conclusions of modern probability theory, not only to remember these basic concepts and the basic probability laws including conditions and conclusions, but also deeply to understand the basic principles and ideas of modern probability;

2．to fully master the four basic convergence theorems (Monotone Convergence Theorem, Fatou Lemma, Dominated Convergence Theorem, and Bounded Convergence Theorems) and be able to apply them in many important topics and different problems;

3．to clearly understand the probability meaning, difference, and relationships of several kind of convergence concepts (almost everywhere convergence; convergence in measure/probability; Convergence in Lp Norm; Weak Convergence) and be able to apply them in different problems;

4．to fully master the very important concepts of conditional expectations and conditional probabilities and to improve the ability of solving practical problems by applying the basic probability methods of "conditioning".

5. to clearly understand and master the basic concepts regarding martingales including the existence, uniqueness, properties and applications of martingales, super- and sub-martingales and be able to apply the important martingale method in the study of modern theory of stochastic processes, stochastic analysis and financial mathematics.

完成本课程后，学生应能够：

1. 深入了解和掌握现代概率论的基本概念和结论，不仅需要记住基本的定理中的条件和结论，还需要深入理解现代概率论的基本概念和想法。

2. 完全掌握四种基本的收敛定理（单调收敛定理，Fatou 引理，控制收敛定理，有界收敛定理），并且可以在许多重要的问题上运用他们。

3. 清晰地理解几种不同收敛概念（几乎处处收敛，依概率收敛，Lp 收敛，弱收敛）的概率意义、区别以及关系，并且可以在许多不同的问题上运用他们

4. 完全掌握条件期望、条件概率的概念，提高在实际问题中使用"条件期望"的能力

5. 清晰地理解与掌握鞅的基本概念，包括存在性、唯一性、性质与应用、超鞅与半鞅，能够在随机过程、随即分析与金融数学中运用鞅方法。

17. 课程内容及教学日历 （如授课语言以英文为主，则课程内容介绍可以用英文；如团队教学或模块教学，教学日历须注明主讲人）
Course Contents (in Parts/Chapters/Sections/Weeks. Please notify name of instructor for course section(s), if this is a team teaching or module course.)

Section 1: Independence, Expectation and Convergence (15h): Borel-Cantelli Lemma, Convergence Theorems, Jensen's Inequality for Convex Functions, The Schwarz Inequality, Orthogonal Projection, Holder from Jensen; Convergence in Probability, Weak Convergence, Convergence in Distributions, Characteristic Functions.

Section 2: Conditional probability and conditional expectation (3h): Definition of conditional expectation and conditional probability, Existence & Uniqueness, Properties of conditional expectation and conditional probability, Tower Property, Some Important Inequalities.

Section 3: Martingales (12h): Definition of martingales, Properties of martingales, Super-martingales, Sub-martingales, Examples, Convergence of martingales, Stopping times, Optional Sampling Theorem.

Section 4: Super-martingales and Sub-martingales (8h): Definitions and Properties of Super-martingales and Sub-martingales, Examples, Doob Decomposition. Convergence of martingales, Stopping times, Optional Sampling Theorem.

Section 5: Martingale Inequality and Martingale Convergence Theorems (8h): Uniform Integrability; UI Martingales; Martingale Convergence Theorems; Backwards Martingale Convergence Theorems; Strong Law of Large Numbers; Martingale Central Limit Theorem.

Section 6：Brownian motion: Basic properties. **(2h)**

18. 教材及其它参考资料 Textbook and Supplementary Readings

Textbook: Jean Jacod & Philip Protter，《Probability Essentials》，Springer-Verlag, Berlin Heidelberg.
Supplementary Readings:
1. David Williams，，《Probability with Martingales》，Cambridge University Press, Cambridge, 1991.
2. 严士健，王隽骧，刘秀芳，《概率论基础》，科学出版社

课程评估 ASSESSMENT

19. 评估形式　　　评估时间　　　占考试总成绩百分比　违纪处罚　　　备注

Type of Assessment	Time	% of final score	Penalty	Notes
出勤 Attendance				
课堂表现 Class Performance				
小测验 Quiz				
课程项目 Projects				
平时作业 Assignments		20%		
期中考试 Mid-Term Test		30%		
期末考试 Final Exam		50%		
期末报告 Final Presentation				
其它（可根据需要改写以上评估方式）Others (The above may be modified as necessary)				

20. 　　　　记分方式 GRADING SYSTEM

> ☑ A. 十三级等级制 Letter Grading
> ☐ B. 二级记分制（通过/不通过）Pass/Fail Grading

课程审批 REVIEW AND APPROVAL

21. 本课程设置已经过以下责任人/委员会审议通过
This Course has been approved by the following person or committee of authority

> 数学系课程规划与审核委员会
> Curriculum Planning and Review Committee, Department of Mathematics