课程大纲
COURSE SYLLABUS

1.	课程代码/名称 **Course Code/Title**	**MAT7092 随机过程 Stochastic Processes**
2.	课程性质 **Compulsory/Elective**	**专业课 Elective**
3.	课程学分/学时 **Course Credit/Hours**	3/48
4.	授课语言 **Teaching Language**	中英双语 Chinese/English
5.	授课教师 **Instructor(s)**	温家强，助理教授，数学系 慧园 3 栋 404 室(答疑时间：周三下午 3-6 点) <u>wenjq@sustech.edu.cn</u> WEN Jiaqiang, Assistant Professor, Department of Mathematics Rm. 404, Huiyuan 3 Bldg. (Office hours: Wednesday 3-6 pm)
6.	是否面向本科生开放 **Open to undergraduates or not**	是 YES
7.	先修要求 **Pre-requisites**	（如面向本科生开放，请注明区分内容。 If the course is open to undergraduates, please indicate the difference.） MA215 概率论 Probability Theories
8.	教学目标 **Course Objectives**	

（如面向本科生开放，请注明区分内容。 If the course is open to undergraduates, please indicate the difference.）

随机过程作为概率论的一个重要分支，它是研究随机现象随时间变化规律性的数学工具，是随机系列课程的重要组成部分。它来源于实际，具有深刻的应用背景，它可广泛应用于金融学、经济与管理科学、信息科学、生物科学、计算机科学以及其他工程技术领域。随机过程本身也是今后学习随机分析和数理金融的重要基础。

本课程将从概率论的基本概念开始，包括随机变量，数学期望和极限定理；然后，将依次介绍泊松过程、更新理论、离散时间马尔科夫链、连续时间马尔科夫链、鞅的概念和布朗运动与其他马尔科夫过程。

学习完本课程后，学生应了解并掌握随机过程的基本概念和结论； 掌握条件期望的定义及其性质，掌握泊松过程的几种等价定义，掌握更新过程和极限定理，能够熟练的判断每个离散时间马尔科夫链的类型，能根据实际问题分析连续时间马尔科夫链的齐次性和马氏性，掌握鞅和停时的定义以及收敛定理，掌握布朗运动的定义和简单性质，了解其他马尔科夫过程的定义和相关概念。

As an important branch of probability theory, random process is a mathematical tool for studying the regularity of random phenomena over time, and an important part of the random series of courses. It comes from reality and has a profound application background. It can be widely used in finance, economics and management science, information science, biological science, computer science and other engineering technology fields. The stochastic process itself is also an important foundation for studying stochastic analysis and mathematical finance in the future.

This course will start with the basic concepts of probability theory, including random variables, mathematical expectations and limit theorems; then, it will introduce the concepts of Poisson process, update

theory, discrete-time Markov chain, continuous-time Markov chain, and martingale in turn And Brownian motion and other Markov processes.

After completing this course, students should understand and master the basic concepts and conclusions of stochastic processes; master the definition and nature of conditional expectations, master several equivalent definitions of Poisson processes, master the update process and limit theorem, and be able to make judgments proficiently Each type of discrete-time Markov chain can analyse the homogeneity and Markov property of continuous-time Markov chain according to actual problems, grasp the definition of martingale and stopping time and the convergence theorem, and grasp the definition and simple properties of Brownian motion , Understand the definitions and related concepts of other Markov processes.

9.	教学方法 **Teaching Methods**

（如面向本科生开放，请注明区分内容。 If the course is open to undergraduates, please indicate the difference.）

理论课程，课堂讲授为主
Theoretical courses, mainly in teaching

10.	教学内容 **Course Contents** （如面向本科生开放，请注明区分内容。 If the course is open to undergraduates, please indicate the difference.）

Section 1 （4 hours）	**准备知识**：概率、随机变量举例、数学期望、特征函数和概率极限定理、随机过程 **Introduction**: probability, examples of random variables, mathematical expectations, characteristic functions and probability limit theorems, random processes
Section 2 （8 hours）	**Poisson 过程**：计数过程和泊松过程、泊松流、复合泊松过程、条件泊松过程 **Poisson process**: counting process and Poisson process, Poisson flow, compound Poisson process, conditional Poisson process
Section 3 （8 hours）	**更新理论**：N(t)的分布、极限定理、更新定理及其应用、延迟更新过程、更新报酬过程、再现过程、平稳点过程 **Renewal theory**: distribution of N(t), limit theorem, renewal theorem and its application, delayed renewal process, renewal reward process, recurrence process, stable point process
Section 4 （8 hours）	**离散时间 Markov 链**：马氏链及其转移概率、Chapman-Kolmogorov 方程、K-C 方程、极限定理、类之间的转移和赌徒破产问题、分支过程、Markov 链的应用、时间可逆的 Markov 链、半 Markov 过程 **Discrete-time Markov chain**: Markov chain and its transition probability, Chapman-Kolmogorov equation, KC equation, limit theorem, transfer between classes and gambler bankruptcy problems, branching process, Markov chain application, time reversible Markov chain, semi-Markov process
Section 5 （8 hours）	**连续时间 Markov 链**：连续时间马氏链、泊松过程是马氏链、生灭过程、Kolmogorov 微分方程、极限概率、时间可逆性、倒向链 排队论的应用、一致化 **Continuous-time Markov chain**: continuous-time Markov chain, Poisson process is Markov chain, birth and death process, Kolmogorov differential equation, limit probability, time reversibility, reverse chain Application of queuing theory, unification

Section 6 （4 hours）	鞅：鞅、停时、鞅的 Azuma 不等式、下鞅上鞅和鞅收敛定理、推广的 Azuma 不等式
Section 7 （8 hours）	**Brown 运动与其他 Markov 过程**：布朗运动、布朗运动的简单性质、漂移布朗运动、向后向前扩散过程、布朗桥与经验过程、极限分布、Markov 散粒噪声过程、平稳过程 **Brownian motion and other Markov processes**: Brownian motion, simple properties of Brownian motion, drifting Brownian motion, backward and forward diffusion process, Brownian bridge and empirical process, limit distribution, Markov shot noise process, stationary process

11.	课程考核 **Course Assessment**
	（①考核形式 Form of examination；②.分数构成 grading policy；③如面向本科生开放，请注明区分内容。If the course is open to undergraduates, please indicate the difference.） 平时作业 **Assignments 25%** 期中考试 **Mid-Term Test 25%** 期末考试 **Final Exam 50%**

12.	教材及其它参考资料 **Textbook and Supplementary Readings**
	1. Stochastic Processes, 2th edition, Sheldon M. 著 2. 随机过程，何书元，编著