南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

课程详述

COURSE SPECIFICATION

以下课程信息可能根据实际授课需要或在课程检讨之后产生变动。如对课程有任何疑问，请联系授课教师。

The course information as follows may be subject to change, either during the session because of unforeseen circumstances, or following review of the course at the end of the session. Queries about the course should be directed to the course instructor.

1.	课程名称 Course Title	泛函分析（研究生） **Functional Analysis（PG）**
2.	授课院系 **Originating Department**	数学系 Mathematics
3.	课程编号 **Course Code**	MAT7003
4.	课程学分 **Credit Value**	3
5.	课程类别 **Course Type**	专业选修课 Major Elective Courses （请保留相应选项 **Please only keep the relevant information**）
6.	授课学期 **Semester**	秋季 Fall
7.	授课语言 **Teaching Language**	英文 English （请保留相应选项 **Please only keep the relevant information**）
8.	授课教师、所属学系、联系方式（如属团队授课，请列明其他授课教师） **Instructor(s), Affiliation& Contact** **（For team teaching, please list all instructors）**	Raul Ures，教授 Raul Ures, Professor
9.	实验员/助教、所属学系、联系方式 **Tutor/TA(s), Contact**	无 NA （请保留相应选项 **Please only keep the relevant information**）
10.	选课人数限额(可不填) **Maximum Enrolment（Optional）**	

		讲授 Lectures	习题/辅导/讨论 Tutorials	实验/实习 Lab/Practical	其它(请具体注明) Other（Please specify）	总学时 Total
11.	授课方式 **Delivery Method**					
	学时数 **Credit Hours**	48				48

12.	先修课程、其它学习要求 **Pre-requisites or Other Academic Requirements**	泛函分析（本科）（MA302） Functional Analysis (MA302)
13.	后续课程、其它学习规划 **Courses for which this course is a pre-requisite**	
14.	其它要求修读本课程的学系 **Cross-listing Dept.**	

教学大纲及教学日历 SYLLABUS

15. **教学目标 Course Objectives**

本课程是本科泛函分析课程的继续与深入，着重介绍有重要应用价值的经典理论，为学生的其他研究生数学课程和相关的科研工作打下基础。

This course is a continuation of the undergraduate course “Functional Analysis”. It emphasizes the classical theories that have important applications, laying a foundation for other related graduate courses and research.

16. **预达学习成果 Learning Outcomes**

本课程强调抽象理论和具体应用的结合。通过一学期的学习，学生应掌握 Hahn-Banach 定理，并了解在正线性泛函的扩展的应用，以及在凸规划问题的应用。对弱拓扑的有基本的掌握。掌握紧算子和 Fredholm 算子相关内容，并了解在二阶椭圆算子和非局部扩散算子上的应用。通过学习一般的谱定理，加深对线性代数中有限维谱定理的认识，通过学习有限维和无限维谱定理的差异，进一步认识无限维空间。并通过对有界对称，正规，酉算子以及无界自伴算子的谱理论的学习，对谱理论有更深入的认识。通过学习本课程，还会为一些后续学习做铺垫，如通过对谱定理尤其是对希尔伯特空间上谱定理的学习，为学生在量子力学的数学描述打下了基础。

This course is a balanced mix of abstract theories and applications. Through one semester of study, students should be skilled at the Hahn-Banach theorem and understand the application of extensions in positive linear functionals and the application of convex programming problems. And have a basic grasp of weak topology. Students should grasp the contents of compact operators and Fredholm operators, and understand the application of second-order elliptic operators and non-local diffusion operators. By learning the general spectral theorem, the understanding of the finite-dimensional spectral theorem in linear algebra is deepened, and the infinite dimensional space is further understood by learning the difference between the finite-dimensional and infinite-dimensional spectral theorems. And through the study of the spectral theory of bounded symmetry, normal, unitary operator and unbounded self-adjoint operator, student will have a deeper understanding in spectral theory. Through the study of this course, it will also lay the foundation for some follow-up learning, such as the study of the spectral theorem, especially the spectral theorem in Hilbert space, which lays a foundation for the mathematical description of quantum mechanics.

17. 课程内容及教学日历（如授课语言以英文为主，则课程内容介绍可以用英文；如团队教学或模块教学，教学日历须注明主讲人）
Course Contents (in Parts/Chapters/Sections/Weeks. Please notify name of instructor for course section(s), if this is a team teaching or module course.)

1. Hahn-Banach Theorem
   1.1. The extension theorem
   1.2. Hyperplane separation of convex sets
   1.3. Applications
      1.3.1 Extension of positive linear functionals
      1.3.2 Lagrange multipliers of convex programming problems
2. Weak and weak * topologies
   2.1. Weak convergence and weak compactness of unit ball in reflexive Banach spaces
   2.2. Weak* convergence and weak* sequential compactness—Helly's Theorem
   2.3. Banach-Alaoglu Theorem
   2.4. Applications
      2.4.1. Approximation of the delta-function by continuous functions
      2.4.2. Approximate quadrature
      2.4.3. Existence of PDE via Galerkin's method
3. General spectral theory
   3.1. Spectral radius and Gelfand's theorem
   3.2. Functional calculus, spectral mapping theorem
   3.3. Spectral decomposition/separation theorem
   3.4. Isolated eigenvalues
      3.4.1 Algebraic multiplicity
      3.4.2 Laurent expansion of the resolvent operator near isolated eigenvalue
      3.4.3 Stability of a finite set of isolated eigenvalues under small operator perturbation
   3.5. Spectrum of the adjoint operator
   3.6. The case of unbounded but closed operators
4. Compact operators and Fredholm operators
   4.1. Riesz-Schauder theory
   4.2. Hilbert-Schmidt theorem, min-max characterization of eigenvalues
   4.3. Positive compact operators: Krein-Rutman theorem (for the special case of Banach space C(Q), where Q is a compact Hausdorff space)
   4.4. Fredholm operators
      4.4.1 Characterization of Fredholm operators, pseudoinverse
      4.4.2 Fredholm index: index of product of two operators, constancy of index under small or compact perturbation
      4.4.3 Essential spectrum of a bounded operator, and its constancy under compact perturbation
   4.5. Applications
      4.5.1. Second order elliptic operators
      4.5.2. Non-local diffusion operators
      4.5.3. Toeplitz operators
5. Spectral theory of bounded symmetric, normal and unitary operators
   5.1. The spectrum of symmetric operators
   5.2. Functional calculus for symmetric operators
   5.3. Spectral resolution of symmetric operators
   5.4. Absolutely continuous, singular, and point spectra
   5.5. The spectral representation of symmetric operators
   5.6. Spectral resolution of normal operators
   5.7. Spectral resolution of unitary operators
   5.8. Examples
6. Unbounded self-adjoint operators
   6.1. Spectral resolution via Cayley transform
   6.2. The extension of unbounded symmetric operators, deficiency indices
   6.3. The Friedrichs extension
   6.4. Examples
7. Semigroups of operators
   7.1. Strongly continuous one-parameter semigroups
   7.2. The generation of semigroups: Hille-Yosida theorem
   7.3. Exponential decay of semigroups
   7.4. Examples: semigroups defined by parabolic equations, and by nonlocal diffusion equations

18. 教材及其它参考资料 **Textbook and Supplementary Readings**

1.Functional Analysis, by Peter Lax.

2.Perturbation Theory for Linear Operators, by T. Kato.

3.泛函分析讲义（上、下），张恭庆等编著

课程评估 ASSESSMENT

19.

评估形式 Type of Assessment	评估时间 Time	占考试总成绩百分比 % of final score	违纪处罚 Penalty	备注 Notes
出勤 Attendance				
课堂表现 Class Performance				
小测验 Quiz				
课程项目 Projects				
平时作业 Assignments		30		
期中考试 Mid-Term Test		30		
期末考试 Final Exam		40		
期末报告 Final Presentation				
其它（可根据需要改写以上评估方式） Others (The above may be modified as necessary)				

20. **记分方式 GRADING SYSTEM**

☑ A. 十三级等级制 **Letter Grading**
☐ B. 二级记分制（通过/不通过） **Pass/Fail Grading**

课程审批 REVIEW AND APPROVAL

21. 本课程设置已经过以下责任人/委员会审议通过
This Course has been approved by the following person or committee of authority
