南方科技大学 SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

课程详述

COURSE SPECIFICATION

以下课程信息可能根据实际授课需要或在课程检讨之后产生变动。如对课程有任何疑问，请联系授课教师。

The course information as follows may be subject to change, either during the session because of unforeseen circumstances, or following review of the course at the end of the session. Queries about the course should be directed to the course instructor.

1.	课程名称 Course Title	测度论与积分（研究生） Measure Theory and Integration（PG）
2.	授课院系 Originating Department	数学系 Department of Mathematics
3.	课程编号 Course Code	MAT7002
4.	课程学分 Credit Value	3
5.	课程类别 Course Type	专业选修课 Major Elective Courses
6.	授课学期 Semester	秋季 Fall
7.	授课语言 Teaching Language	中英双语 English & Chinese
8.	授课教师、所属学系、联系方式（如属团队授课，请列明其他授课教师） Instructor(s), Affiliation& Contact （For team teaching, please list all instructors）	数学系 王学锋 教授 wangxf@sustc.edu.cn 88018754 慧园 3 栋 530 数学系 苏琳琳 助理教授 sull@sustc.edu.cn 88018679 慧园 3 栋 403 Department of Mathematics; Xuefeng Wang, Professor; Room 530, Block 3, Wisdom Valley Department of Mathematics; Linlin Su, Assistant Professor; Room 403, Block 3, Wisdom Valley
9.	实验员/助教、所属学系、联系方式 Tutor/TA(s), Contact	待公布 To be announced
10.	选课人数限额(可不填) Maximum Enrolment （Optional）	

11. 授课方式 Delivery Method	讲授 Lectures	习题/辅导/讨论 Tutorials	实验/实习 Lab/Practical	其它(请具体注明) Other（Please specify）	总学时 Total
学时数 Credit Hours	48				48

12.	先修课程、其它学习要求 **Pre-requisites or Other Academic Requirements**	实变函数论(MA301) Theory of Functions of a Real Variable (MA301)
13.	后续课程、其它学习规划 **Courses for which this course is a pre-requisite**	研究生应用数学课程和研究生偏微分方程课程 graduate applied mathematics, graduate PDE course
14.	其它要求修读本课程的学系 **Cross-listing Dept.**	

教学大纲及教学日历 SYLLABUS

15. **教学目标 Course Objectives**

本课是大学实变函数课程的继续。在大学课程里，学生已经掌握了实轴上的 Lebesgue 测度及积分理论，故而自然地本课将以抽象的测度论开始，再讲抽象可测空间上的积分理论，L^p 空间等。这些内容为其它研究生课程如概率论打下基础。课程的最后部分把学生重新带回 R^n 空间中，学习调和分析的一些最基本的内容，包括傅氏变换的有界性和傅里叶级数的收敛性等等广泛应用于应用数学和偏微分方程的内容。

This course is a continuation of the undergraduate Functions of a Real Variable (MA301), in which the students already learn Lebesgue measure and Lebesgue integration theory. Therefore, it is natural for this course to start with abstract measure theory, then integration theory on abstract spaces, such as L^p spaces; these materials will prepare the students well for their further graduate courses, such as probability theory. The last part of this course returns to R^n space, some basic facts from harmonic analysis, including the bounded property of the Fourier transform and the convergence of the Fourier series, etc., which are widely used in applied mathematics and partial differential equations.

16. **预达学习成果 Learning Outcomes**

通过学习掌握抽象可测空间上的积分理论，L^p 空间的性质和调和分析的一些基本内容。为后续课程的学习打下良好的分析基础。

Through learning to grasp the integral theory on the abstract measurable space, the nature of L^p space, and some basic contents of harmonic analysis. To lay a good analysis foundation for the follow-up course.

17. **课程内容及教学日历 （如授课语言以英文为主，则课程内容介绍可以用英文；如团队教学或模块教学，教学日历须注明主讲人）**
Course Contents (in Parts/Chapters/Sections/Weeks. Please notify name of instructor for course section(s), if this is a team teaching or module course.)

1. General Measure Spaces: Their Properties and Construction （抽象可测空间的性质与构造）(10 credit hours)
   - 1.1. Measures and Measurable Sets （测度与可测集合）
   - 1.2. Signed Measures: The Hahn and Jordan Decompositions （变号测度，Hahn 分解和 Jordan 分解）
   - 1.3. The Caratheodory Measure Induced by an Outer Measure （外测度诱导的 Caratheodory 测度）
   - 1.4. The Construction of Outer Measures （外测度的构造）
   - 1.5. The Caratheodory-Hahn Theorem: The Extension of a Premeasure to a Measure （Caratheodory-Hahn 定理：预测度的扩展）
2. Integration Over General Measure Spaces （抽象测度空间上的积分） (10 credit hours)
   - 2.1. Measurable Functions （可测函数）
   - 2.2. Integration of Nonnegative Measurable Functions （非负可测函数的积分）
   - 2.3. Integration of General Measurable Functions （一般可测函数的积分）
   - 2.4. The Radon-Nikodym Theorem （Radon-Nikodym 定理）
3. The Construction of Particular Measures （特殊测度的构造） (8 credit hours)
   - 3.1. Product Measures: The Theorems of Fubini and Tonelli （乘积测度：Fubini 定理和 Tonelli 定理）
   - 3.2. Lebesgue Measure on Euclidean Space R" （n 维欧式空间上的 Lebesgue 测度）
   - 3.3. Cumulative Distribution Functions on R and Lebesgue-Stieltjes integral （1 维欧式空间上的累积分布函数和 Lebesgue-Stieltjes 积分）
4. General L^p Spaces: Completeness, Duality, and Weak Convergence （抽象的 L^p 空间，完备性，对偶空间，弱收敛）(12 credit hours)
   - 4.1. The Completeness of $L^p(X, \mu)$ （$L^p(X, \mu)$ 的完备性）
   - 4.2. The Riesz Representation Theorem for the Dual of $L^p(X, \mu)$, $1 \le p < \infty$ （$L^p(X, \mu)$, $1 \le p < \infty$ 对偶空间的 Riesz 表示定理）
   - 4.3. The Kantorovitch Representation Theorem for the Dual of $L^\infty(X, \mu)$ （$L^\infty(X, \mu)$ 对偶空间的 Kantorovitch 表示定理）
   - 4.4. Weak Sequential Compactness in $L^1(X, \mu)$: The Dunford-Pettis Theorem （$L^1(X, \mu)$的弱列紧性：Dunford-Pettis 定理）
5. Some Basics in Harmonic Analysis （一些调和分析基础知识） (8 credit hours)
   - 5.1. The Fourier transform on L^1 and L^2 （L^1 和 L^2 上的傅里叶变换）
   - 5.2. Riesz-Thorin interpolation theorem and the Fourier transform on L^p, $1<p<2$ （Riesz-Thorin 插值定理及 L^p, $1<p<2$ 上的傅里叶变换）
   - 5.3. The Marcinkiewicz interpolation theorem （Marcinkiewicz 插值定理）
   - 5.4. Hardy-Littlewood maximal function and Hardy-Littlewood maximal inequality（Hardy-Littlewood 极大函数和 Hardy-Littlewood 极大不等式）
   - 5.5. Hardy-Littlewood-Sobolev inequality（Hardy-Littlewood-Sobolev 不等式）
   - 5.6. Trigonometric Fourier Series （傅里叶三角函数级数）

18. 教材及其它参考资料 Textbook and Supplementary Readings

1. Real Analysis, fourth edition, by Halsey L. Royden and Patrick M. Fitzpatrick, 2010.
2. Measure and Integral, R. Wheeden and A. Zygmund, 1997.

课程评估 ASSESSMENT

19.

评估形式 Type of Assessment	评估时间 Time	占考试总成绩百分比 % of final score	违纪处罚 Penalty	备注 Notes
出勤 Attendance				
课堂表现 Class Performance				
小测验 Quiz				
课程项目 Projects				
平时作业 Assignments		40%		
期中考试 Mid-Term Test		20%		
期末考试 Final Exam		40%		
期末报告 Final				

Presentation				
其它（可根据需要改写以上评估方式） **Others (The above may be modified as necessary)**				

20. **记分方式 GRADING SYSTEM**

☑ A. **十三级等级制 Letter Grading** ☐ B. **二级记分制（通过/不通过） Pass/Fail Grading**

课程审批 REVIEW AND APPROVAL

21. **本课程设置已经过以下责任人/委员会审议通过**
This Course has been approved by the following person or committee of authority
