南方科技大学 SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

课程详述

COURSE SPECIFICATION

以下课程信息可能根据实际授课需要或在课程检讨之后产生变动。如对课程有任何疑问，请联系授课教师。

The course information as follows may be subject to change, either during the session because of unforeseen circumstances, or following review of the course at the end of the session. Queries about the course should be directed to the course instructor.

1.	课程名称 Course Title	初等数论 Elementary Number Theory
2.	授课院系 Originating Department	数学系 Department of Mathematics
3.	课程编号 Course Code	MA209-16
4.	课程学分 Credit Value	3
5.	课程类别 Course Type	专业选修课 Major Elective Courses
6.	授课学期 Semester	秋季 Fall
7.	授课语言 Teaching Language	中英双语 English & Chinese
8.	授课教师、所属学系、联系方式（如属团队授课，请列明其他授课教师） Instructor(s), Affiliation& Contact （For team teaching, please list all instructors）	胡勇，数学系 慧园 3 栋 409 huy@sustc.edu.cn 0755-8801-5910 Yong Hu, Department of Mathematics Block 3, Room 409, Wisdom Valley huy@sustc.edu.cn 0755-8801-5910
9.	实验员/助教、所属学系、联系方式 Tutor/TA(s), Contact	待公布 To be announced
10.	选课人数限额(可不填) Maximum Enrolment （Optional）	

11. 授课方式 Delivery Method	讲授 Lectures	习题/辅导/讨论 Tutorials	实验/实习 Lab/Practical	其它(请具体注明) Other（Please specify）	总学时 Total
学时数 Credit Hours	48				48

12.	先修课程、其它学习要求 **Pre-requisites or Other Academic Requirements**	数学分析 II(MA102a)或高等数学 A 下(MA102B), 线性代数 II (MA104b) Mathematical Analysis II (MA102a) or Advanced Mathematics A II(MA102B), Linear Algebra II (MA104b)
13.	后续课程、其它学习规划 **Courses for which this course is a pre-requisite**	抽象代数 Abstract Algebra
14.	其它要求修读本课程的学系 **Cross-listing Dept.**	

教学大纲及教学日历 SYLLABUS

15. **教学目标 Course Objectives**

课程主要讲述整除理论，素数，同余方程，典型的丢番图方程，指数与原根，二次剩余与二次互反律，积性数论函数、Dirichlet 级数、格点和 Minkowski 定理等基础数论知识，以及这些理论的一些实际应用。学生通过本课程学习可以打下良好的数论知识基础，体会该学科的魅力，了解数论的应用，为后续更高级课程的学习做好知识准备。

Main topics of the course include：basic theories about divisibility, primes, congruences, special Diophantine equations, index and primitive roots, quadratic residues and reciprocity, multiplicative arithmetic functions, Dirichlet series, latticen and Minkowski's theorem, as well as some important applications of number theory. Students are expected to lay down a solid background in number theory, have a feeling of the beauty of the subject, learn about its applications, and get well prepared for subsequent, more advanced courses.

16. **预达学习成果 Learning Outcomes**

通过对本课程的学习，学生能够理解和掌握初等数论的基本理论和一些重要应用。同时，学生应当理解现代数学(包括代数和分析等)在经典的数论问题中的应用，加深对现代数论前沿研究内容和研究方法的理解。

An adequate training through this course should help the students to understand the basic methods and techniques in elementary number theory as well as some important applications. Also, students are expected to have a good understanding of the roles of modern mathematics - including algebra and analysis - in the applications of classical number theoretic questions, thus enhancing their comprehensions of advanced research topics and methods in the frontiers of modern number theory.

17. **课程内容及教学日历 （如授课语言以英文为主，则课程内容介绍可以用英文；如团队教学或模块教学，教学日历须注明主讲人）**
Course Contents (in Parts/Chapters/Sections/Weeks. Please notify name of instructor for course section(s), if this is a team teaching or module course.)

第一章 整数的整除性 (8)

§ 1 自然数集与数学归纳法

1.1 良序原理和取整函数

1.2 数学归纳法

§ 2 整除和素数基本理论

2.1 整除的定义与基本性质

2.2 整数的不同进制表示

2.3 最大公因子和辗转相除法

2.4 素数与算术基本定理

§ 3 整数的 p-进赋值

3.1 p-进赋值

3.2 n! 的素因子分解

§ 4 Pythagoras 三元组

4.1 初等方法求解

4.2 单位圆周上的有理点

4.3 费马的无穷递降法

第二章 同余及其应用 (12h)

§ 1 同余概述

1.1 同余类和剩余系

1.2 线性同余方程

§ 2 中国剩余定理

2.1 定理的陈述及证明

2.2 剩余类环

2.3 既约剩余系

2.4 特殊同余式

2.5 中国剩余定理的环论表述

§3 一些应用和补充

3.1 素性检验

3.2 循环赛赛制

3.3 伪素数

第三章 原根及其应用 (8h)

§1 模运算中整数的阶

1.1 整数剩余类的阶

1.2 素数的原根

§2 存在原根的正整数

2.1 素数幂情形

2.2 一般情形

§3 原根的一些应用

3.1 用整数的阶做素性检验

3.2 通用指数和 Carmichael 数

3.3 离散对数和幂剩余

第四章 二次剩余 (6 学时)

§1 二次剩余与非剩余

§2 二次互反律

§3 应用：零知识证明

第五章 数论函数与 Dirichlet 级数 (12 学时)

§1 数论函数

1.1 积性函数

1.2 Dirichlet 卷积和 Mobius 反演公式

§2 Dirichlet 级数

2.1 形式级数和 Euler 乘积

2.2 Dirichlet 特征和 L 函数

§ 3 Dirichlet 级数确定的函数

3.1 Dirichlet 级数的收敛性

3.2 Dirichlet L 函数和等差数列中的素数

3.3 Riemann Zeta 函数和 Riemann 猜想简介

第六章 格与 Minkowski 定理 (2 学时)

§ 1 格点和 Minkowski 定理

§ 2 Minkowski 定理的应用

2.1 平方和

2.2 Dirichlet 逼近定理

Chapter 1 Divisibility of Integers (8h)

§ 1 The set of integers and mathematical induction

1.1 The well ordering property and the integral part

1.2 Mathematical induction

§ 2 Divisibility and prime numbers

2.1 Divisibility of integers

2.2 Representations of integers in different bases

2.3 Greatest common divisor and Euclidean algorithm

2.4 Prime numbers and the fundamental theorem of arithmetic

§ 3 P-adic valuation of integers

3.1 The p-adic valuation

3.2 Factorization of n!

§ 4 Pythagorean triples

4.1 Finding solutions with elementary methods

4.2 Rational point on the unit circle

4.3 Fermat's method of infinite descent

Chapter 2 Congruences and Applications (12h)

§ 1 Introduction to congruences

1.1 Congruences and systems of residues

1.2 Linear congruences

§ 2 The Chinese Remainder Theorem

2.1 Statement and proof of the theorem

2.2 The ring of congruence classes

2.3 Reduced residue systems

2.4 Some special congruences

2.5 Ring theoretic interpretation of the Chinese remainder theorem

§ 3 Some applications and complements

3.1 Divisibility tests

3.2 Round-Robin tournaments

3.3 Pseudo primes

Chapter 3 Primitive Roots and Applications (8h)

§ 1 Order of integers in modular arithmetic

1.1 The order of an integer residue class

1.2 Primitive roots for primes

§ 2 Numbers having primitive roots

2.1 Prime powers

2.2 The general case

§ 3 Applications of primitive roots

3.1 Primality tests using orders of integers

3.2 Universal exponents and Carmichael numbers

3.3 Discrete logarithms and power residues

Chapter 4 Quadratic Residues (6h)

§ 1 Quadratic residues and nonresidues

§ 2 The law of quadratic reciprocity

§ 3 An application: Zero-knowledge proof

Chapter 5 Arithmetic Functions and Dirichlet Series (12h)

§ 1 Arithmetic functions

1.1 Multiplicative functions

1.2 Dirichlet product and Möbius Inversion

§ 2 Dirichlet series

2.1 Formal series and Euler products

2.2 Dirichlet characters and L-functions

§ 3 Functions defined by Dirichlet series

3.1 Convergence of Dirichlet series

3.2 Dirichlet L-functions and primes in arithmetic progressions

3.3 Complements on the Riemann zeta function and the Riemann hypothesis

Chapter 6 Lattices and Minkowski's theorem (2h)

§ 1 Lattice points and Minkowski's theorem

§ 2 Applications of Minkowski's theorem

2.1 Sums of squares

2.2 Dirichlet's approximation theorem

18. 教材及其它参考资料 Textbook and Supplementary Readings

教材 Textbook:

Kenneth H. ROSEN, Elementary Number Theory and its applications (6th edition) , ISBN-13: 978-0321500311 / ISBN-10: 0321500318

推荐参考书 Supplementary Readings:

Tom M. Apostol , Introduction to Analytic Number Theory (Undergraduate Texts in Mathematics) , ISBN-13: 978-1441928054 / ISBN-10: 1441928057

潘承洞、潘承彪著，《简明数论》，北京大学出版社，ISBN: 9787301035283

课程评估 ASSESSMENT

19.

评估形式 Type of Assessment	评估时间 Time	占考试总成绩百分比 % of final score	违纪处罚 Penalty	备注 Notes
出勤 Attendance				
课堂表现 Class Performance		10		
小测验 Quiz				
课程项目 Projects				
平时作业 Assignments		20		
期中考试 Mid-Term Test		35		
期末考试 Final Exam		35		
期末报告 Final Presentation				
其它（可根据需要改写以上评估方式） Others (The above may be modified as necessary)				

20. 记分方式 GRADING SYSTEM

☑ A. 十三级等级制 Letter Grading
☐ B. 二级记分制（通过/不通过） Pass/Fail Grading

课程审批 REVIEW AND APPROVAL

21. 本课程设置已经过以下责任人/委员会审议通过
This Course has been approved by the following person or committee of authority