课程详述

COURSE SPECIFICATION

以下课程信息可能根据实际授课需要或在课程检讨之后产生变动。如对课程有任何疑问，请联系授课教师。

The course information as follows may be subject to change, either during the session because of unforeseen circumstances, or following review of the course at the end of the session. Queries about the course should be directed to the course instructor.

1.	课程名称 Course Title	高等代数 II Advanced Linear Algebra II
2.	授课院系 Originating Department	数学系 Department of Mathematics
3.	课程编号 Course Code	
4.	课程学分 Credit Value	4
5.	课程类别 Course Type	专业基础课 Major Foundational Courses
6.	授课学期 Semester	春季 Spring
7.	授课语言 Teaching Language	中英双语 English & Chinese
8.	授课教师、所属学系、联系方式（如属团队授课，请列明其他授课教师） Instructor(s), Affiliation& Contact（For team teaching, please list all instructors）	胡勇，数学系 慧园 3 栋 409 huy@sustech.edu.cn 0755-8801-5910 Yong Hu, Department of Mathematics Block 3, Room 409, Wisdom Valley huy@sustech.edu.cn 0755-8801-5910
9.	实验员/助教、所属学系、联系方式 Tutor/TA(s), Contact	待公布 To be announced
10.	选课人数限额(可不填) Maximum Enrolment（Optional）	

11.	授课方式 Delivery Method	讲授 Lectures	习题/辅导/讨论 Tutorials	实验/实习 Lab/Practical	其它(请具体注明) Other（Please specify）	总学时 Total
	学时数 Credit Hours	64	32			96

12.	先修课程、其它学习要求 **Pre-requisites or Other Academic Requirements**	高等代数 I / Advanced Linear Algebra I
13.	后续课程、其它学习规划 **Courses for which this course is a pre-requisite**	初等数论、抽象代数 Elementary Number Theory, Abstract Algebra
14.	其它要求修读本课程的学系 **Cross-listing Dept.**	

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教学大纲及教学日历 SYLLABUS

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15. 教学目标 Course Objectives

本课程主要为数学系数学与应用数学专业设计，分成 I、II 两部分，按两个学期连贯教学。课程旨在引导学生深入系统地学习该专业所需要的代数学基本知识，为数学专业高年级的后续课程打下扎实的基础。课程内容将按照高于同类课程的标准进行教学和考核，以培养出代数学基础最扎实的学生为目标。

本课程(高等代数 II)内容主要包括：双线性型和内积空间，多项式分解及其应用，线性算子的谱理论，Jordan 标准形，欧氏空间的旋转变换群，以及若干选讲专题。

This course is primarily designed for students majored in pure and applied mathematics, and is divided into Parts I and II, lasting ideally for two consecutive semesters. It aims at leading students into systematic and thorough studies of the fundamentals of modern algebra, thus getting them to lay a solid foundation for subsequent, more advanced courses in math major. The contents of the course and the standards of assessment will normally surpass the other courses in the same series, the objective being to foster students with best background in algebra.

Main topics of this course (Advanced Linear Algebra II) include：Bilinear forms and inner product spaces, polynomial factorization and applications, spectral theory of linear operators, Jordan canonical forms, rotation group of Euclidean spaces, and some selected topics.

16. 预达学习成果 Learning Outcomes

通过对本课程的学习，学生能够理解和掌握高等代数的更深入理论(包括内积空间、若当标准形、正规算子、谱定理等)和在一些重要应用。同时，学生应当逐渐培养出较好的抽象思维能力和进行严密数学推理和证明的能力。

An adequate training through this course should help the students to understand the basics of advanced linear algebra (such as inner product spaces, Jordan canonical forms, normal operators, spectral theorems) as well as some important applications. Also, students are expected to gradually foster the ability of abstract thinking and doing logically rigorous arguments and proofs in math.

17. 课程内容及教学日历 （如授课语言以英文为主，则课程内容介绍可以用英文；如团队教学或模块教学，教学日历须注明主讲人）
Course Contents (in Parts/Chapters/Sections/Weeks. Please notify name of instructor for course section(s), if this is a team teaching or module course.)

3.2 线性算子代入多项式取值

3.3 化零多项式和极小多项式

3.4 线性算子的上三角化

第三章 线性算子谱理论 (14h)

§1 内积空间上的算子

1.1 自伴算子及其矩阵

1.2 正规算子及其矩阵

§2 谱定理

2.1 酉空间上的正规算子

2.2 Euclid 空间上的自伴算子

2.3 矩阵的正交相似和酉相似

§3 正定算子和等距变换

3.1 正定算子及其矩阵

3.2 等距变换及其矩阵

3.3 极分解和方阵奇异值

第四章 Jordan 标准形 (16 学时)

§1 广义特征子空间和准对角形

1.1 定义和基本性质

1.2 复算子的广义特征子空间分解

1.3 特征多项式与 Cayley-Hamilton 定理

1.4 复算子的准对角形

§2 幂零算子

2.1 幂零算子和幂零矩阵

2.2 复算子和复矩阵的 Jordan 分解

2.3 循环子空间分解

§3 复算子的 Jordan 标准形

3.1 Jordan 块与 Jordan 标准形

3.2 存在和唯一性

3.3 实用算法和计算举例

第五章 欧氏空间的旋转变换群 (6 学时)

§1 欧氏空间的正规算子

1.1 实正规算子的标准形

1.2 欧氏空间的等距变换

§2 低维空间中的旋转

第六章 专题选讲 (4 学时)

§1 商空间与正合列

§2 多重线性代数

§3 矩阵分析

§4 矩阵群和李代数

Chapter 0 Some Reviews (2h)

§1 Vector spaces and linear maps

§2 Change of basis and transformations of matrices

§3 Invariant subspaces and eigenspaces

Chapter 1 Bilinear Forms and Inner Product Spaces (14h)

§1 Bilinear and hermitian forms

1.1 Bilinear forms and their matrices

1.2 Symmetric bilinear forms and quadratic forms

1.3 Hermitian forms and hermitian matrices

§2 Dual spaces and adjoint maps

2.1 Linear functionals and dual space

2.2 Orthogonal relations and dual bases

2.3 Orthogonal and unitary similarity of matrices

§ 3 Positive operators and isometries

3.1 Positive operators and their matrices

3.2 Isometries and their matrices

3.3 Polar decomposition and singular values of square matrices

Chapter 4 Jordan Canonical Forms (16h)

§ 1 Generalized eigenspaces and quasi-diagonal forms

1.1 Definition and basic properties

1.2 Decomposition into generalized eigenspaces for complex operators

1.3 Characteristic polynomial and Cayley-Hamilton theorem

1.4 Quasi-diagonal form of complex operators

§ 2 Nilpotent operators

2.1 Nilpotent operators and nilpotent matrices

2.2 Jordan decomposition of complex operators and matrices

2.3 Decomposition inot cyclic subspaces

§ 3 Jordan canonical form of complex operators

3.1 Jordan blocks and Jordan canonical forms

3.2 Existence and uniqueness

3.3 Practical algorithm and computational examples

Chapter 5 Rotation Group of Euclidean Spaces (6h)

§ 1 Normal operators on Euclidean spaces

1.1 Canonical form of real normal operators

1.2 Isometries of Euclidean spaces

§ 2 Rotations in low dimensional spaces

Chapter 6 Selected Topics (4h)

§ 1 Quotient spaces and exact sequences

§ 2 Multilinear algebra

§ 3 Matrix analysis

§ 4 Matrix groups and Lie algebras

18. **教材及其它参考资料 Textbook and Supplementary Readings**

教材 Textbook:

蓝以中，《高等代数简明教程》下册，第二版，北京大学出版社，2007，ISBN：9787301055793

推荐参考书 Supplementary Readings:

蓝以中，《高等代数学习指南》，北京大学出版社，2008，ISBN: 9787301129050;

李尚志，《线性代数》(数学专业用)，高等教育出版社，2006，ISBN: 9787040198706;

Sheldon Axler, Linear Algebra Done Right, 3rd edition, Springer, 2015, ISBN: 9783319110790;

Peter D. Lax, Linear Algebra and its applications, 2nd edition, 2007, ISBN 9780471751564

课程评估 ASSESSMENT

19.	评估形式 Type of Assessment	评估时间 Time	占考试总成绩百分比 % of final score	违纪处罚 Penalty	备注 Notes
	出勤 Attendance				
	课堂表现 Class Performance		5		
	小测验 Quiz		15		
	课程项目 Projects				
	平时作业 Assignments		10		

期中考试 Mid-Term Test		30		
期末考试 Final Exam		40		
期末报告 Final Presentation				
其它（可根据需要改写以上评估方式） Others (The above may be modified as necessary)				

20.   记分方式 GRADING SYSTEM

> ☑ A. 十三级等级制 Letter Grading
> ☐ B. 二级记分制（通过/不通过） Pass/Fail Grading

课程审批 REVIEW AND APPROVAL

21.   本课程设置已经过以下责任人/委员会审议通过
        This Course has been approved by the following person or committee of authority