An Introduction to maSAE

Andreas Dominik Cullmann

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1 Introduction

Superscripts For partially exhaustive auxiliary information, Mandallaz ([1, p. 1023], [2, p. 383f] defines $Z^t(x) = Z^{(1)t}(x) + Z^{(2)t}(x)$ whereas Hill [3, p. 4 and p. 18] defines $Z^t(x) = Z^{(0)t}(x) + Z^{(1)t}(x)$. I will stick with Mandallaz' notation, changing $Z^{(0)t}(x)$ to $Z^{(1)t}(x)$ in Hill's formulae!

Indicies Mandallaz and Hill inconsistently uses the indices $_2$ and $_{s_2}$, they really both denote the same: the set s_2. For the sets s_0 and s_1 they consistsently use $_0$ and $_1$. I have change all set indices to $s_{[012]}$.

Hill uses $\bar{Z}^{(1)}_{0,G}$ (and $\bar{Z}^{(1)}_0$ which ([3, p. 18]) is the exact mean). So I do drop the index, which is misleadingly refering to some set (and I do so for $\bar{\mathcal{Z}}^{(1)}_{0,G}$).

Mandallaz uses $\bar{\hat{R}}_{2,G}$ when calculating the variance of the residuals in G, for example in a2.26, where $\bar{\hat{R}}_{2,G}$ is clearly $\bar{\hat{R}}(x)$ while summing over s_2 and G. I use the latter form.

References I reference [4] as a1, [5] as a2, [6] as b1, [1] as b2, [7] as c1, [2] as c2 and [3] as h.

Estimators In tables 1 and 2 in the first block there are always the synthetic, the small area and Mandallaz' extended estimator for different kinds of auxiliary information: exhaustive, non-exhaustive, partially exhaustive. In the second block there's the estimators for three-phase partially exhaustive, three-phase non-exhaustive . Table 2 gives the clustered versions of the estimators in table 1.

Tables 3 and 4 give the same information in a more compact way, I have replaced the empirical mean and variance of the Residuals in G for clustered sampling,

$$\frac{\sum_{x \in s_2,G} M(x)\hat{R}_c(x)}{\sum_{x \in s_2,G} M(x)}$$

and

$$\frac{1}{n_{s_2,G} - 1} \sum_{x \in s_2,G} \left(\frac{M(x)}{\bar{M}(x)}\right)^2 (\hat{R}_c(x) - \bar{\hat{R}}_c(x))^2,$$

by their shorter notations $\bar{\hat{R}}_{c,s_2,G}(x)$ and $\hat{V}(\hat{R}_{c,s_2,G}(x))$ and likewise for unclustered sampling.

Looking at their third blocks (partially exhaustive auxiliary information), we see that the estimators and variances are identical for two- and three-phase sampling. Yet partially exhaustive auxiliary information is what [3, p. 22] uses. To me it seems as useless as exhaustive auxiliary information in three-phase, which boils down to two-phase with more observations!

cl	s	ext	exh	small	ref	formula
no	2p	no	yes	no	a2.18	$\hat{Y}_{G,synth} = \bar{Z}_G^t \hat{\beta}_{s_2}$
-	-	-	-	-	a2.19	$\hat{V}(\times) = \bar{Z}_G^t \hat{\Sigma}_{\hat{\beta}_{s_2}} \bar{Z}_G$
no	2p	no	yes	yes	a2.20	$\hat{Y}_{G,small} = \hat{Y}_{G,synth} + \frac{1}{n_{s_2,G}} \sum_{x \in s_2,G} \hat{R}(x)$
-	-	-	-	-	a2.21	$\hat{V}(\times) \approx \hat{V}\left(\hat{Y}_{G,synth}\right) + \frac{1}{n_{s_2,G}} \frac{1}{n_{s_2,G}-1} \sum_{x \in s_2,G} \left(\hat{R}(x) - \bar{\hat{R}}(x)\right)^2$
no	2p	yes	yes	no	a2.31	$\hat{\hat{Y}}_{G,synth} = \bar{\mathcal{Z}}_G^t \hat{\theta}_{s_2}$
-	-	-	-	-	a2.33	$\hat{V}(\times) = \bar{\mathcal{Z}}_G^t \hat{\Sigma}_{\hat{\theta}_{s_2}} \bar{\mathcal{Z}}_G$
no	2p	no	no	no	a2.22	$\hat{Y}_{G,psynth} = \hat{\bar{Z}}_{s_1,G}^t \hat{\beta}_{s_2}$
-	-	-	-	-	a2.23	$\hat{V}(\times) = \hat{\bar{Z}}_{s_1,G}^t \hat{\Sigma}_{\hat{\beta}_{s_2}} \hat{\bar{Z}}_{s_1,G} + \hat{\beta}_{s_2}^t \hat{\Sigma}_{\hat{\bar{Z}}_{s_1,G}} \hat{\beta}_{s_2}$
no	2p	no	no	yes	a2.25	$\hat{Y}_{G,psmall} = \hat{Y}_{G,psynth} + \frac{1}{n_{s_2,G}} \sum_{x \in s_2,G} \hat{R}(x)$
-	-	-	-	-	a2.26	$\hat{V}(\times) \approx \hat{V}\left(\hat{Y}_{G,synth}\right) + \frac{1}{n_{s_2,G}} \frac{1}{n_{s_2,G}-1} \sum_{x \in s_2,G} \left(\hat{R}(x) - \bar{\hat{R}}(x)\right)^2$
no	2p	yes	no	no	a2.35	$\hat{\hat{Y}}_{G,psynth} = \hat{\bar{\mathcal{Z}}}_{s_1,G}^t \hat{\theta}_{s_2}$
-	-	-	-	-	a2.36	$\hat{V}(\times) = \hat{\bar{\mathcal{Z}}}_{s_1,G}^t \hat{\Sigma}_{\hat{\theta}_{s_2}} \hat{\bar{\mathcal{Z}}}_{s_1,G} + \hat{\theta}_{s_2}^t \hat{\Sigma}_{\hat{\bar{\mathcal{Z}}}_{s_1,G}} \hat{\theta}_{s_2}$
no	2p	no	part	no	b2.34	$\hat{Y}_{psynth,G,greg} = \left(\bar{Z}_G^{(1)} - \hat{\bar{Z}}_{s_1,G}^{(1)}\right) \hat{\alpha}_{s_2} + \hat{\bar{Z}}_{s_1,G}^t \hat{\beta}_{s_2}$
-	-	-	-	-	b2.35	$\hat{V}(\times) = \frac{n_{s_2}}{n_{s_1}} \bar{Z}_G^{(1)t} \hat{\Sigma}_{\hat{\alpha}_{s_2}} \bar{Z}_G^{(1)} + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{Z}}_{s_1,G}^t \hat{\Sigma}_{\hat{\beta}_{s_2}} \hat{\bar{Z}}_{s_1,G}$
no	2p	no	part	yes	b2.24	$\hat{Y}_{G,greg} = \hat{Y}_{psynth,G,greg} + \sum_{x \in s_2,G} \hat{R}(x)$
-	-	-	-	-	b2.23	$\hat{V}(\times) \approx \hat{V}\left(\hat{Y}_{psynth,G,greg}\right) + \frac{1}{n_{s_2,G}} \frac{1}{n_{s_2,G}-1} \sum_{x \in s_2,G} \left(\hat{R}(x) - \bar{\hat{R}}_{s_2,G}\right)^2$
no	2p	yes	part	no	b2.30	$\hat{\hat{Y}}_{G,greg} = \left(\bar{\mathcal{Z}}_G^{(1)} - \hat{\bar{\mathcal{Z}}}_{s_1,G}^{(1)}\right) \hat{\gamma}_{s_2} + \hat{\bar{\mathcal{Z}}}_{s_1,G}^t \hat{\theta}_{s_2}$
-	-	-	-	-	b2.31	$\hat{V}(\times) = \frac{n_{s_2}}{n_{s_1}} \bar{\mathcal{Z}}_G^{(1)t} \hat{\Sigma}_{\hat{\gamma}_{s_2}} \bar{\mathcal{Z}}_G^{(1)} + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{\mathcal{Z}}}_{s_1,G}^t \hat{\Sigma}_{\hat{\theta}_{s_2}} \hat{\bar{\mathcal{Z}}}_{s_1,G}$

cl	s	ext	exh	small	ref	formula
no	3p	no	part	no	h.26a	$\hat{Y}_{G,synth,3p} = \left(\bar{Z}^{(1)}_{G} - \hat{\bar{Z}}^{(1)}_{s_1,G}\right)\hat{\alpha}_{s_2} + \hat{\bar{Z}}^{t}_{s_1,G}\hat{\beta}_{s_2}$
-	-	-	-	-	h.26c	$\hat{V}\,(\times) = \frac{n_{s_2}}{n_{s_1}}\hat{\bar{Z}}^{(1)t}_{s_0,G}\hat{\Sigma}_{\hat{\alpha}_{s_2}}\hat{\bar{Z}}^{(1)}_{s_0,G} + (1-\frac{n_{s_2}}{n_{s_1}})\hat{\bar{Z}}^{t}_{s_1,G}\hat{\Sigma}_{\hat{\beta}_{s_2}}\hat{\bar{Z}}_{s_1,G}$
no	3p	no	part	yes	h.22a	$\hat{Y}_{G,small,3p} = \hat{Y}_{G,synth,3p} + \frac{1}{n_{s_2,G}}\sum_{x\in s_{2,G}}\hat{R}(x)$
-	-	-	-	-	h.23a	$\hat{V}\,(\times) \approx \hat{V}\left(\hat{Y}_{G,synth,3p}\right) + \frac{1}{n_{s_2,G}}\frac{1}{n_{s_2,G}-1}\sum_{x\in s_{2,G}}\left(\hat{R}(x)-\hat{\bar{R}}(x)\right)^2$
no	3p	yes	part	no	h.26a ext	$\hat{\hat{Y}}_{G,extsynth,3p} = \left(\bar{\mathcal{Z}}^{(1)}_{G} - \hat{\bar{\mathcal{Z}}}^{(1)}_{s_1,G}\right)\hat{\gamma}_{s_2} + \hat{\bar{\mathcal{Z}}}^{t}_{s_1,G}\hat{\theta}_{s_2}$
-	-	-	-	-	h.26c ext	$\hat{V}\,(\times) = \frac{n_{s_2}}{n_{s_1}}\hat{\bar{\mathcal{Z}}}^{(1)t}_{s_0,G}\hat{\Sigma}_{\hat{\gamma}_{s_2}}\hat{\bar{\mathcal{Z}}}^{(1)}_{s_0,G} + (1-\frac{n_{s_2}}{n_{s_1}})\hat{\bar{\mathcal{Z}}}^{t}_{s_1,G}\hat{\Sigma}_{\hat{\theta}_{s_2}}\hat{\bar{\mathcal{Z}}}_{s_1,G}$
no	3p	no	no	no	h.26b	$\hat{Y}_{G,psynth,3p} = \left(\hat{\bar{Z}}^{(1)}_{s_0,G} - \hat{\bar{Z}}^{(1)}_{s_1,G}\right)\hat{\alpha}_{s_2} + \hat{\bar{Z}}^{t}_{s_1,G}\hat{\beta}_{s_2}$
-	-	-	-	-	h.26d	$\hat{V}\,(\times) = \hat{\alpha}^{t}_{s_2}\hat{\Sigma}_{\hat{\bar{Z}}^{(1)}_{s_0,G}}\hat{\alpha}_{s_2} + \frac{n_{s_2}}{n_{s_1}}\hat{\bar{Z}}^{(1)t}_{s_0,G}\hat{\Sigma}_{\hat{\alpha}_{s_2}}\hat{\bar{Z}}^{(1)}_{s_0,G} + (1-\frac{n_{s_2}}{n_{s_1}})\hat{\bar{Z}}^{t}_{s_1,G}\hat{\Sigma}_{\hat{\beta}_{s_2}}\hat{\bar{Z}}_{s_1,G}$
no	3p	no	no	yes	h.22b	$\hat{Y}_{G,psmall,3p} = \hat{Y}_{G,psynth,G,3p} + \frac{1}{n_{s_2,G}}\sum_{x\in s_{2,G}}\hat{R}(x)$
–	-	-	-	-	h.23b	$\hat{V}\,(\times) \approx \hat{V}\left(\hat{\hat{Y}}_{G,psynth,3p}\right) + \frac{1}{n_{s_2,G}}\frac{1}{n_{s_2,G}-1}\sum_{x\in s_{2,G}}\left(\hat{R}(x)-\hat{\bar{R}}(x)\right)^2$
no	3p	yes	no	no	c2.23	$\hat{\hat{Y}}_{G,g3reg} = \left(\hat{\bar{\mathcal{Z}}}^{(1)}_{s_0,G} - \hat{\bar{\mathcal{Z}}}^{(1)}_{s_1,G}\right)\hat{\gamma}_{s_2} + \hat{\bar{\mathcal{Z}}}^{t}_{s_1,G}\hat{\theta}_{s_2}$
-	-	-	-	-	c2.24	$\hat{V}\,(\times) = \hat{\gamma}^{t}_{s_2}\hat{\Sigma}_{\hat{\bar{\mathcal{Z}}}^{(1)}_{s_0,G}}\hat{\gamma}_{s_2} + \frac{n_{s_2}}{n_{s_1}}\hat{\bar{\mathcal{Z}}}^{(1)t}_{s_0,G}\hat{\Sigma}_{\hat{\gamma}_{s_2}}\hat{\bar{\mathcal{Z}}}^{(1)}_{s_0,G} + (1-\frac{n_{s_2}}{n_{s_1}})\hat{\bar{\mathcal{Z}}}^{t}_{s_1,G}\hat{\Sigma}_{\hat{\theta}_{s_2}}\hat{\bar{\mathcal{Z}}}_{s_1,G}$

Table 1: Predictors for unclustered sampling, *cl* denotes clustering (yes/no), *s* denotes sampling (2p is two-phase, 3p is three-phase), *ext* denotes using the extended estimator (yes/no), *exh* denotes exhaustiveness of auxiliary information (yes/no/part, the latter meaning partially exhaustive auxiliary information), *small* denotes the area estimator.

cl	s	ext	exh	small	ref	formula
yes	2p	no	yes	no	analogy	$\hat{Y}_{c,G,synth} = \bar{Z}^t_G \hat{\beta}_{c,s_2}$
-	-	-	-	-	analogy	$\hat{V}\left(\times\right) = \bar{Z}^t_G \hat{\Sigma}_{\hat{\beta}_{s_2}} \bar{Z}_G$
yes	2p	no	yes	yes	analogy	$\hat{Y}_{c,G,small} = \hat{Y}_{c,G,synth} + \sum_{x \in s_2,G} M(x) \hat{R}_c(x) / \sum_{x \in s_2,G} M(x)$
-	-	-	-	-	analogy	$\hat{V}(\times) = \hat{V}\left(\hat{Y}_{c,G,synth}\right) + \frac{1}{n_{s_2,G}} \frac{1}{n_{s_2,G}-1} \sum_{x \in s_2,G} \left({M(x)}/{\bar{M}(x)}\right)^2 \left(\hat{R}_c(x) - \bar{\hat{R}}_c(x)\right)^2$
yes	2p	yes	yes	no	a2.48	$\hat{\hat{Y}}_{c,G,synth} = \bar{\mathcal{Z}}^t_G \hat{\theta}_{c,s_2}$
-	-	-	-	-	a2.49	$\hat{V}\left(\times\right) = \bar{\mathcal{Z}}^t_G \hat{\Sigma}_{\hat{\theta}_{c,s_2}} \bar{\mathcal{Z}}_G$
yes	2p	no	no	no	a2.42	$\hat{Y}_{c,G,psynth} = \hat{\bar{Z}}^1_{c,s_1,G} \hat{\beta}_{c,s_2}$
-	-	-	-	-	a2.43	$\hat{V}(\times) = \hat{\bar{Z}}^t_{c,s_1,G} \hat{\Sigma}_{\hat{\beta}_{c,s_2}} \hat{\bar{Z}}_{c,s_1,G} + \hat{\beta}^t_{c,s_2} \hat{\Sigma}_{\hat{\bar{Z}}_{c,s_1,G}} \hat{\beta}_{c,s_2}$
yes	2p	no	no	yes	a2.44	$\hat{Y}_{c,G,psmall} = \hat{Y}_{c,G,psynth} + \sum_{x \in s_2,G} M(x) \hat{R}_c(x) / \sum_{x \in s_2,G} M(x)$
-	-	-	-	-	a2.45	$\hat{V}(\times) = \hat{V}\left(\hat{Y}_{c,G,psynth}\right) + \frac{1}{n_{s_2,G}} \frac{1}{n_{s_2,G}-1} \sum_{x \in s_2,G} \left({M(x)}/{\bar{M}(x)}\right)^2 \left(\hat{R}_c(x) - \bar{\hat{R}}_c(x)\right)^2$
yes	2p	yes	no	no	a2.46	$\hat{\hat{Y}}_{c,G,psynth} = \hat{\bar{\mathcal{Z}}}^t_{c,s_1,G} \hat{\theta}_{c,s_2}$
-	-	-	-	-	a2.47	$\hat{V}\left(\times\right) = \hat{\bar{\mathcal{Z}}}^t_{c,s_1,G} \hat{\Sigma}_{\hat{\theta}_{c,s_2}} \hat{\bar{\mathcal{Z}}}_{c,s_1,G} + \hat{\theta}^t_{c,s_2} \hat{\Sigma}_{\hat{\bar{\mathcal{Z}}}_{c,s_1,G}} \hat{\theta}_{c,s_2}$
yes	2p	no	part	no	analogy	$\hat{Y}_{c,psynth,G,greg} = \left(\bar{Z}^{(1)}_G - \hat{\bar{Z}}^{(1)}_{c,s_1,G}\right) \hat{\alpha}_{c,2} + \hat{\bar{Z}}^t_{c,s_1,G} \hat{\beta}_{c,s_2}$
-	-	-	-	-	analogy	$\hat{V}\left(\times\right) = \frac{n_{s_2}}{n_{s_1}} \bar{Z}^{(1)t}_G \hat{\Sigma}_{\hat{\alpha}_{c,s_2}} \bar{Z}^{(1)}_G + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{Z}}^t_{c,s_1,G} \hat{\Sigma}_{\hat{\beta}_{c,s_2}} \hat{\bar{Z}}_{c,s_1,G}$
yes	2p	no	part	yes	analogy	$\hat{Y}_{c,G,greg} = \hat{Y}_{c,G,psynth} + \sum_{x \in s_2,G} M(x) \hat{R}_c(x) / \sum_{x \in s_2,G} M(x)$
-	-	-	-	-	analogy	$\hat{V}\left(\times\right) = \hat{V}\left(\hat{Y}_{c,psynth,G,greg}\right) + \frac{1}{n_{s_2,G}} \frac{1}{n_{s_2,G}-1} \sum_{x \in s_2,G} \left({M(x)}/{\bar{M}(x)}\right)^2 \left(\hat{R}_c(x) - \bar{\hat{R}}_c(x)\right)^2$
yes	2p	yes	part	no	b1.50	$\hat{\hat{Y}}_{c,G,greg} = \left(\bar{\mathcal{Z}}^{(1)}_G - \hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_1,G}\right) \hat{\gamma}_{c,2} + \hat{\bar{\mathcal{Z}}}^t_{c,s_1,G} \hat{\theta}_{c,2}$
-	-	-	-	-	b1.52	$\hat{V}\left(\times\right) = \frac{n_{s_2}}{n_{s_1}} \bar{\mathcal{Z}}^{(1)t}_G \hat{\Sigma}_{\hat{\gamma}_{c,s_2}} \bar{\mathcal{Z}}^{(1)}_G + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{\mathcal{Z}}}^t_{c,s_1,G} \hat{\Sigma}_{\hat{\theta}_{c,s_2}} \hat{\bar{\mathcal{Z}}}_{c,s_1,G}$

cl	s	ext	exh	small	ref	formula
yes	3p	no	part	no	analogy	$\hat{Y}_{c,G,synth,3p} = \left(\bar{Z}^{(1)}_{G} - \hat{\bar{Z}}^{(1)}_{c,s_1,G}\right)\hat{\alpha}_{c},2 + \hat{\bar{Z}}^{t}_{c,s_1,G}\hat{\beta}_{c,s_2}$
-	-	-	-	-	analogy	$\hat{V}(\times) = \frac{n_{s_2}}{n_{s_1}}\hat{\bar{Z}}^{(1)t}_{c,s_0,G}\hat{\Sigma}_{\hat{\alpha}_{c,s_2}}\hat{\bar{Z}}^{(1)}_{c,s_0,G} + (1-\frac{n_{s_2}}{n_{s_1}})\hat{\bar{Z}}^{t}_{c,s_1,G}\hat{\Sigma}_{\hat{\beta}_{c,s_2}}\hat{\bar{Z}}_{c,s_1,G}$
yes	3p	no	part	yes	analogy	$\hat{Y}_{c,G,small,3p} = \hat{Y}_{c,G,synth,3p} + \sum_{x\in s_{2,G}} M(x)\hat{R}_c(x) / \sum_{x\in s_{2,G}} M(x)$
-	-	-	-	-	analogy	$\hat{V}(\times) \approx \hat{V}\left(\hat{Y}_{c,G,synth,3p}\right) + \frac{1}{n_{s_2,G}}\frac{1}{n_{s_2,G}-1}\sum_{x\in s_2,G} \left(M(x)/\bar{M}(x)\right)^2 (\hat{R}_c(x) - \hat{\bar{R}}_c(x))^2$
yes	3p	yes	part	no	analogy	$\hat{\hat{Y}}_{c,G,extsynth,3p} = \left(\bar{\mathcal{Z}}^{(1)}_{G} - \hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_1,G}\right)\hat{\gamma}_{c,2} + \hat{\bar{\mathcal{Z}}}^{t}_{c,s_1,G}\hat{\theta}_{c,2}$
-	-	-	-	-	analogy	$\hat{V}(\times) = \frac{n_{s_2}}{n_{s_1}}\hat{\bar{\mathcal{Z}}}^{(1)t}_{c,s_0,G}\hat{\Sigma}_{\hat{\gamma}_{c,s_2}}\hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_0,G} + (1-\frac{n_{s_2}}{n_{s_1}})\hat{\bar{\mathcal{Z}}}^{t}_{c,s_1,G}\hat{\Sigma}_{\hat{\theta}_{c,s_2}}\hat{\bar{\mathcal{Z}}}_{c,s_1,G}$
yes	3p	no	no	no	analogy	$\hat{Y}_{G,psynth,3p} = \left(\hat{\bar{Z}}^{(1)}_{c,s_0,G} - \hat{\bar{Z}}^{(1)}_{c,s_1,G}\right)\hat{\alpha}_{c,2} + \hat{\bar{Z}}^{t}_{c,s_1,G}\hat{\beta}_{c,s_2}$
-	-	-	-	-	analogy	$\hat{V}(\times) = \hat{\alpha}^{t}_{c,s_2}\hat{\Sigma}_{\hat{\bar{Z}}^{(1)}_{c,s_0,G}}\hat{\alpha}_{c,s_2} + \frac{n_{s_2}}{n_{s_1}}\hat{\bar{Z}}^{(1)t}_{c,s_0,G}\hat{\Sigma}_{\hat{\alpha}_{c,s_2}}\hat{\bar{Z}}^{(1)}_{c,s_0,G} + (1-\frac{n_{s_2}}{n_{s_1}})\hat{\bar{Z}}^{t}_{c,s_1,G}\hat{\Sigma}_{\hat{\beta}_{c,s_2}}\hat{\bar{Z}}_{c,s_1,G}$
yes	3p	no	no	yes	analogy	$\hat{Y}_{c,G,psmall,3p} = \hat{Y}_{G,psynth,G,3p} + \sum_{x\in s_{2,G}} M(x)\hat{R}_c(x) / \sum_{x\in s_{2,G}} M(x)$
-	-	-	-	-	analogy	$\hat{V}(\times) \approx \hat{V}\left(\hat{\hat{Y}}_{c,G,psynth,3p}\right) + \frac{1}{n_{s_2,G}}\frac{1}{n_{s_2,G}-1}\sum_{x\in s_2,G} \left(M(x)/\bar{M}(x)\right)^2 (\hat{R}_c(x) - \hat{\bar{R}}_c(x))^2$
yes	3p	yes	no	no	c1.53	$\hat{\hat{Y}}_{c,G,g3reg} = \left(\hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_0,G} - \hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_1,G}\right)\hat{\gamma}_{c,2} + \hat{\bar{\mathcal{Z}}}^{t}_{c,s_1,G}\hat{\theta}_{c,2}$
-	-	-	-	-	c1.55	$\hat{V}(\times) = \hat{\gamma}^{t}_{c,s_2}\hat{\Sigma}_{\hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_0,G}}\hat{\gamma}_{c,s_2} + \frac{n_{s_2}}{n_{s_1}}\hat{\bar{\mathcal{Z}}}^{(1)t}_{c,s_0,G}\hat{\Sigma}_{\hat{\gamma}_{c,s_2}}\hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_0,G} + (1-\frac{n_{s_2}}{n_{s_1}})\hat{\bar{\mathcal{Z}}}^{t}_{c,s_1,G}\hat{\Sigma}_{\hat{\theta}_{c,s_2}}\hat{\bar{\mathcal{Z}}}_{c,s_1,G}$

Table 2: Predictors for clustered sampling, *cl* denotes clustering (yes/no), *s* denotes sampling (2p is two-phase, 3p is three-phase), *ext* denotes using the extended estimator (yes/no), *exh* denotes exhaustiveness of auxiliary information (yes/no/part, the latter meaning partially exhaustive auxiliary information), *small* denotes the area estimator.

2p	3p
$\hat{Y}_{G,synth} = \bar{Z}_G^t \hat{\beta}_{s_2}$ $\hat{V}(\times) = \bar{Z}_G^t \hat{\Sigma}_{\hat{\beta}_{s_2}} \bar{Z}_G$	- -
$\hat{Y}_{G,small} = \hat{Y}_{G,synth} + \bar{\hat{R}}_{s_2,G}(x)$ $\hat{V}(\times) \approx \hat{V}\left(\hat{Y}_{G,synth}\right) + \frac{1}{n_{s_2,G}} \hat{V}(\hat{R}_{s_2,G}(x))$	- -
$\hat{\hat{Y}}_{G,synth} = \bar{\mathcal{Z}}_G^t \hat{\theta}_{s_2}$ $\hat{V}(\times) = \bar{\mathcal{Z}}_G^t \hat{\Sigma}_{\hat{\theta}_{s_2}} \bar{\mathcal{Z}}_G$	- -
$\hat{Y}_{G,psynth} = \hat{\bar{Z}}_{s_1,G}^t \hat{\beta}_{s_2}$ $\hat{V}(\times) = \hat{\bar{Z}}_{s_1,G}^t \hat{\Sigma}_{\hat{\beta}_{s_2}} \hat{\bar{Z}}_{s_1,G} + \hat{\beta}_{s_2}^t \hat{\Sigma}_{\hat{\bar{Z}}_{s_1,G}} \hat{\beta}_{s_2}$	$\hat{Y}_{G,psynth,3p} = \left(\hat{\bar{Z}}_{s_0,G}^{(1)} - \hat{\bar{Z}}_{s_1,G}^{(1)}\right) \hat{\alpha}_{s_2} + \hat{\bar{Z}}_{s_1,G}^t \hat{\beta}_{s_2}$ $\hat{V}(\times) = \hat{\alpha}_{s_2}^t \hat{\Sigma}_{\hat{\bar{Z}}_{s_0,G}^{(1)}} \hat{\alpha}_{s_2} + \frac{n_{s_2}}{n_{s_1}} \hat{\bar{Z}}_{s_0,G}^{(1)t} \hat{\Sigma}_{\hat{\alpha}_{s_2}} \hat{\bar{Z}}_{s_0,G}^{(1)} + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{Z}}_{s_1,G}^t \hat{\Sigma}_{\hat{\beta}_{s_2}} \hat{\bar{Z}}_{s_1,G}$
$\hat{Y}_{G,psmall} = \hat{Y}_{G,psynth} + \bar{\hat{R}}_{s_2,G}(x)$ $\hat{V}(\times) \approx \hat{V}\left(\hat{Y}_{G,synth}\right) + \frac{1}{n_{s_2,G}} \hat{V}(\hat{R}_{s_2,G}(x))$	$\hat{Y}_{G,psmall,3p} = \hat{Y}_{G,psynth,G,3p} + \bar{\hat{R}}_{s_2,G}(x)$ $\hat{V}(\times) \approx \hat{V}\left(\hat{\hat{Y}}_{G,psynth,3p}\right) + \frac{1}{n_{s_2,G}} \hat{V}(\hat{R}_{s_2,G}(x))$
$\hat{\hat{Y}}_{G,psynth} = \hat{\bar{\mathcal{Z}}}_{s_1,G}^t \hat{\theta}_{s_2}$ $\hat{V}(\times) = \hat{\bar{\mathcal{Z}}}_{s_1,G}^t \hat{\Sigma}_{\hat{\theta}_{s_2}} \hat{\bar{\mathcal{Z}}}_{s_1,G} + \hat{\theta}_{s_2}^t \hat{\Sigma}_{\hat{\bar{\mathcal{Z}}}_{s_1,G}} \hat{\theta}_{s_2}$	$\hat{\hat{Y}}_{G,g3reg} = \left(\hat{\bar{\mathcal{Z}}}_{s_0,G}^{(1)} - \hat{\bar{\mathcal{Z}}}_{s_1,G}^{(1)}\right) \hat{\gamma}_{s_2} + \hat{\bar{\mathcal{Z}}}_{s_1,G}^t \hat{\theta}_{s_2}$ $\hat{V}(\times) = \hat{\gamma}_{s_2}^t \hat{\Sigma}_{\hat{\bar{\mathcal{Z}}}_{s_0,G}^{(1)}} \hat{\gamma}_{s_2} + \frac{n_{s_2}}{n_{s_1}} \hat{\bar{\mathcal{Z}}}_{s_0,G}^{(1)t} \hat{\Sigma}_{\hat{\gamma}_{s_2}} \hat{\bar{\mathcal{Z}}}_{s_0,G}^{(1)} + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{\mathcal{Z}}}_{s_1,G}^t \hat{\Sigma}_{\hat{\theta}_{s_2}} \hat{\bar{\mathcal{Z}}}_{s_1,G}$
$\hat{Y}_{psynth,G,greg} = \left(\bar{Z}_G^{(1)} - \hat{\bar{Z}}_{s_1,G}^{(1)}\right) \hat{\alpha}_{s_2} + \hat{\bar{Z}}_{s_1,G}^t \hat{\beta}_{s_2}$ $\hat{V}(\times) = \frac{n_{s_2}}{n_{s_1}} \bar{Z}_G^{(1)t} \hat{\Sigma}_{\hat{\alpha}_{s_2}} \bar{Z}_G^{(1)} + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{Z}}_{s_1,G}^t \hat{\Sigma}_{\hat{\beta}_{s_2}} \hat{\bar{Z}}_{s_1,G}$	$\hat{Y}_{G,synth,3p} = \left(\bar{Z}_G^{(1)} - \hat{\bar{Z}}_{s_1,G}^{(1)}\right) \hat{\alpha}_{s_2} + \hat{\bar{Z}}_{s_1,G}^t \hat{\beta}_{s_2}$ $\hat{V}(\times) = \frac{n_{s_2}}{n_{s_1}} \hat{\bar{Z}}_{s_0,G}^{(1)t} \hat{\Sigma}_{\hat{\alpha}_{s_2}} \hat{\bar{Z}}_{s_0,G}^{(1)} + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{Z}}_{s_1,G}^t \hat{\Sigma}_{\hat{\beta}_{s_2}} \hat{\bar{Z}}_{s_1,G}$
$\hat{Y}_{G,greg} = \hat{Y}_{psynth,G,greg} + \bar{\hat{R}}_{s_2,G}(x)$ $\hat{V}(\times) \approx \hat{V}\left(\hat{Y}_{psynth,G,greg}\right) + \frac{1}{n_{s_2,G}} \hat{V}(\hat{R}_{s_2,G}(x))$	$\hat{Y}_{G,small,3p} = \hat{Y}_{G,synth,3p} + \bar{\hat{R}}_{s_2,G}(x)$ $\hat{V}(\times) \approx \hat{V}\left(\hat{Y}_{G,synth,3p}\right) + \frac{1}{n_{s_2,G}} \hat{V}(\hat{R}_{s_2,G}(x))$
$\hat{\hat{Y}}_{G,greg} = \left(\bar{\mathcal{Z}}_G^{(1)} - \hat{\bar{\mathcal{Z}}}_{s_1,G}^{(1)}\right) \hat{\gamma}_{s_2} + \hat{\bar{\mathcal{Z}}}_{s_1,G}^t \hat{\theta}_{s_2}$ $\hat{V}(\times) = \frac{n_{s_2}}{n_{s_1}} \bar{\mathcal{Z}}_G^{(1)t} \hat{\Sigma}_{\hat{\gamma}_{s_2}} \bar{\mathcal{Z}}_G^{(1)} + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{\mathcal{Z}}}_{s_1,G}^t \hat{\Sigma}_{\hat{\theta}_{s_2}} \hat{\bar{\mathcal{Z}}}_{s_1,G}$	$\hat{\hat{Y}}_{G,extsynth,3p} = \left(\bar{\mathcal{Z}}_G^{(1)} - \hat{\bar{\mathcal{Z}}}_{s_1,G}^{(1)}\right) \hat{\gamma}_{s_2} + \hat{\bar{\mathcal{Z}}}_{s_1,G}^t \hat{\theta}_{s_2}$ $\hat{V}(\times) = \frac{n_{s_2}}{n_{s_1}} \hat{\bar{\mathcal{Z}}}_{s_0,G}^{(1)t} \hat{\Sigma}_{\hat{\gamma}_{s_2}} \hat{\bar{\mathcal{Z}}}_{s_0,G}^{(1)} + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{\mathcal{Z}}}_{s_1,G}^t \hat{\Sigma}_{\hat{\theta}_{s_2}} \hat{\bar{\mathcal{Z}}}_{s_1,G}$

Table 3: Predictors for unclustered sampling, columns show predictors and their variances for two- and three-phase sampling. The three blocks represent exhaustive, non-exhaustive and partially exhaustive auxiliary information, in each block the first parts show the synthetic, the seonds parts the small and the third part the extended small area estimator.

2p	3p
$\hat{Y}_{c,G,synth} = \bar{Z}_G^t \hat{\beta}_{c,s_2}$ $\hat{V}\,(\times) = \bar{Z}_G^t \hat{\Sigma}_{\hat{\beta}_{s_2}} \bar{Z}_G$	- -
$\hat{Y}_{c,G,small} = \hat{Y}_{c,G,synth} + \bar{\hat{R}}_{c,s_2,G}(x)$ $\hat{V}(\times) = \hat{V}\left(\hat{Y}_{c,G,synth}\right) + \frac{1}{n_{s_2,G}} \hat{V}(\hat{R}_{c,s_2,G}(x))$	- -
$\hat{\hat{Y}}_{c,G,synth} = \bar{\mathcal{Z}}_G^t \hat{\theta}_{c,s_2}$ $\hat{V}\,(\times) = \bar{\mathcal{Z}}_G^t \hat{\Sigma}_{\hat{\theta}_{c,s_2}} \bar{\mathcal{Z}}_G$	- -
$\hat{Y}_{c,G,psynth} = \hat{\bar{Z}}^{1}_{c,s_1,G} \hat{\beta}_{c,s_2}$ $\hat{V}(\times) = \hat{\bar{Z}}^{t}_{c,s_1,G} \hat{\Sigma}_{\hat{\beta}_{c,s_2}} \hat{\bar{Z}}_{c,s_1,G} + \hat{\beta}^t_{c,s_2} \hat{\Sigma}_{\hat{\bar{Z}}_{c,s_1,G}} \hat{\beta}_{c,s_2}$	$\hat{Y}_{G,psynth,3p} = \left(\hat{\bar{Z}}^{(1)}_{c,s_0,G} - \hat{\bar{Z}}^{(1)}_{c,s_1,G}\right) \hat{\alpha}_{c,2} + \hat{\bar{Z}}^{t}_{c,s_1,G} \hat{\beta}_{c,s_2}$ $\hat{V}\,(\times) = \hat{\alpha}^t_{c,s_2} \hat{\Sigma}_{\hat{\bar{Z}}^{(1)}_{c,s_0,G}} \hat{\alpha}_{c,s_2} + \frac{n_{s_2}}{n_{s_1}} \hat{\bar{Z}}^{(1)t}_{c,s_0,G} \hat{\Sigma}_{\hat{\alpha}_{c,s_2}} \hat{\bar{Z}}^{(1)}_{c,s_0,G} + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{Z}}^{t}_{c,s_1,G} \hat{\Sigma}_{\hat{\beta}_{c,s_2}} \hat{\bar{Z}}_{c,s_1,G}$
$\hat{Y}_{c,G,psmall} = \hat{Y}_{c,G,psynth} + \bar{\hat{R}}_{c,s_2,G}(x)$ $\hat{V}(\times) = \hat{V}\left(\hat{Y}_{c,G,psynth}\right) + \frac{1}{n_{s_2,G}} \hat{V}(\hat{R}_{c,s_2,G}(x))$	$\hat{Y}_{c,G,psmall,3p} = \hat{Y}_{G,psynth,G,3p} + \bar{\hat{R}}_{c,s_2,G}(x)$ $\hat{V}\,(\times) \approx \hat{V}\left(\hat{\hat{Y}}_{c,G,psynth,3p}\right) + \frac{1}{n_{s_2,G}} \hat{V}(\hat{R}_{c,s_2,G}(x))$
$\hat{\hat{Y}}_{c,G,psynth} = \hat{\bar{\mathcal{Z}}}^{t}_{c,s_1,G} \hat{\theta}_{c,s_2}$ $\hat{V}\,(\times) = \hat{\bar{\mathcal{Z}}}^{t}_{c,s_1,G} \hat{\Sigma}_{\hat{\theta}_{c,s_2}} \hat{\bar{\mathcal{Z}}}_{c,s_1,G} + \hat{\theta}^t_{c,s_2} \hat{\Sigma}_{\hat{\bar{\mathcal{Z}}}_{c,s_1,G}} \hat{\theta}_{c,s_2}$	$\hat{\hat{Y}}_{c,G,g3reg} = \left(\hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_0,G} - \hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_1,G}\right) \hat{\gamma}_{c,2} + \hat{\bar{\mathcal{Z}}}^{t}_{c,s_1,G} \hat{\theta}_{c,2}$ $\hat{V}\,(\times) = \hat{\gamma}^t_{c,s_2} \hat{\Sigma}_{\hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_0,G}} \hat{\gamma}_{c,s_2} + \frac{n_{s_2}}{n_{s_1}} \hat{\bar{\mathcal{Z}}}^{(1)t}_{c,s_0,G} \hat{\Sigma}_{\hat{\gamma}_{c,s_2}} \hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_0,G} + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{\mathcal{Z}}}^{t}_{c,s_1,G} \hat{\Sigma}_{\hat{\theta}_{c,s_2}} \hat{\bar{\mathcal{Z}}}_{c,s_1,G}$
$\hat{Y}_{c,psynth,G,greg} = \left(\bar{Z}^{(1)}_G - \hat{\bar{Z}}^{(1)}_{c,s_1,G}\right) \hat{\alpha}_{c,2} + \hat{\bar{Z}}^{t}_{c,s_1,G} \hat{\beta}_{c,s_2}$ $\hat{V}\,(\times) = \frac{n_{s_2}}{n_{s_1}} \bar{Z}^{(1)t}_G \hat{\Sigma}_{\hat{\alpha}_{c,s_2}} \bar{Z}^{(1)}_G + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{Z}}^{t}_{c,s_1,G} \hat{\Sigma}_{\hat{\beta}_{c,s_2}} \hat{\bar{Z}}_{c,s_1,G}$	$\hat{Y}_{c,G,synth,3p} = \left(\bar{Z}^{(1)}_G - \hat{\bar{Z}}^{(1)}_{c,s_1,G}\right) \hat{\alpha}_{c},2 + \hat{\bar{Z}}^{t}_{c,s_1,G} \hat{\beta}_{c,s_2}$ $\hat{V}\,(\times) = \frac{n_{s_2}}{n_{s_1}} \hat{\bar{Z}}^{(1)t}_{c,s_0,G} \hat{\Sigma}_{\hat{\alpha}_{c,s_2}} \hat{\bar{Z}}^{(1)}_{c,s_0,G} + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{Z}}^{t}_{c,s_1,G} \hat{\Sigma}_{\hat{\beta}_{c,s_2}} \hat{\bar{Z}}_{c,s_1,G}$
$\hat{Y}_{c,G,greg} = \hat{Y}_{c,G,psynth} + \bar{\hat{R}}_{c,s_2,G}(x)$ $\hat{V}\,(\times) = \hat{V}\left(\hat{Y}_{c,psynth,G,greg}\right) + \frac{1}{n_{s_2,G}} \hat{V}(\hat{R}_{c,s_2,G}(x))$	$\hat{Y}_{c,G,small,3p} = \hat{Y}_{c,G,synth,3p} + \bar{\hat{R}}_{c,s_2,G}(x)$ $\hat{V}\,(\times) \approx \hat{V}\left(\hat{Y}_{c,G,synth,3p}\right) + \frac{1}{n_{s_2,G}} \hat{V}(\hat{R}_{c,s_2,G}(x))$
$\hat{\hat{Y}}_{c,G,greg} = \left(\bar{\mathcal{Z}}^{(1)}_G - \hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_1,G}\right) \hat{\gamma}_{c,2} + \hat{\bar{\mathcal{Z}}}^{t}_{c,s_1,G} \hat{\theta}_{c,2}$ $\hat{V}\,(\times) = \frac{n_{s_2}}{n_{s_1}} \bar{\mathcal{Z}}^{(1)t}_G \hat{\Sigma}_{\hat{\gamma}_{c,s_2}} \bar{\mathcal{Z}}^{(1)}_G + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{\mathcal{Z}}}^{t}_{c,s_1,G} \hat{\Sigma}_{\hat{\theta}_{c,s_2}} \hat{\bar{\mathcal{Z}}}_{c,s_1,G}$	$\hat{\hat{Y}}_{c,G,extsynth,3p} = \left(\bar{\mathcal{Z}}^{(1)}_G - \hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_1,G}\right) \hat{\gamma}_{c,2} + \hat{\bar{\mathcal{Z}}}^{t}_{c,s_1,G} \hat{\theta}_{c,2}$ $\hat{V}\,(\times) = \frac{n_{s_2}}{n_{s_1}} \hat{\bar{\mathcal{Z}}}^{(1)t}_{c,s_0,G} \hat{\Sigma}_{\hat{\gamma}_{c,s_2}} \hat{\bar{\mathcal{Z}}}^{(1)}_{c,s_0,G} + (1 - \frac{n_{s_2}}{n_{s_1}}) \hat{\bar{\mathcal{Z}}}^{t}_{c,s_1,G} \hat{\Sigma}_{\hat{\theta}_{c,s_2}} \hat{\bar{\mathcal{Z}}}_{c,s_1,G}$

Table 4: Predictors for clustered sampling, columns show predictors and their variances for two- and three-phase sampling. The three blocks represent exhaustive, non-exhaustive and partially exhaustive auxiliary information, in each block the first parts show the synthetic, the seonds parts the small and the third part the extended small area estimator.

```
> fake_weights <- function(df) {
+     df[["weights"]] <- 1
+     df[["weights"]][df[["x2"]] == 0] <- 0.12
+     return(df)
+ }
> suppressWarnings(rm(s1, s2, s0))
> data("s1", "s2", "s0", package = "maSAE")
> s0$x1 <- s0$x3 <- NULL
> s0 <- fake_weights(s0)
> s1 <- fake_weights(s1)
> s2 <- fake_weights(s2)
> s12 <- maSAE::bind_data(s1, s2)
> s012 <- maSAE::bind_data(s1, s2, s0)
> tm <- data.frame(x1 = c(150, 200), x2 = c(23, 23), x3 = c(7, 7.5), g = c("a", "b"))
> tm_p <- data.frame(x2 = c(23, 23), g = c("a", "b"))
> #% unclustered
>
> ##% un-weighted
> ###% two-phase
> ####% partially exhaustive
> object <- maSAE::saObj(data = s12, f = y ~ x1 + x2 + x3 | g, s2 = "phase2", smallAreaMea
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance  psynth var_psynth   psmall var_psmall
1         a   374.4659 288.8545 362.889   62.83338 374.3667   293.4668
2         b   384.7822 250.1123 378.087   61.99708 384.7325   202.7989

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> ####% exhaustive
> object <- maSAE::saObj(data = s12, f = y ~ x1 + x2 + x3 | g, s2 = "phase2", smallAreaMea
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   365.0818 221.8780 353.5113   40.29034 364.9890   270.9238
2         b   380.5571 144.4836 373.8780   40.03398 380.5234   180.8358

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> ####% non-exhaustive
> object <- maSAE::saObj(data = s12, f = y ~ x1 + x2 + x3 | g, s2 = "phase2")
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   378.8590 487.3680 367.2796   303.1129 378.7573   533.7463
2         b   391.8262 417.3442 385.1562   314.9198 391.8016   455.7217
```

```
> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> ###% three-phase
> object <- maSAE::saObj(data = s012, f = y ~ x1 + x2 + x3 | g, s1 = "phase1", s2 = "phase
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   397.1866 311.8078 385.5963   82.87085 397.0740   313.5043
2         b   404.2197 271.9036 397.5939   82.71266 404.2394   223.5145

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> ##% weighted
> ###% two-phase
> ####% partially exhaustive
> object <- maSAE::saObj(data = s12, f = y ~ x1 + x2 + x3 | g,
+                         s2 = "phase2", smallAreaMeans = tm_p,
+                         auxiliaryWeights = "weights")
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   376.9497 291.8285 365.3586   61.26629 376.8363   291.8997
2         b   389.2490 243.3468 382.4815   60.04105 389.1269   200.8429

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> ####% exhaustive
> object <- maSAE::saObj(data = s12, f = y ~ x1 + x2 + x3 | g,
+                         s2 = "phase2", smallAreaMeans = tm,
+                         auxiliaryWeights = "weights")
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   365.0818 221.8780 353.5113   40.29034 364.9890   270.9238
2         b   380.5571 144.4836 373.8780   40.03398 380.5234   180.8358

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> ####% non-exhaustive
> object <- maSAE::saObj(data = s12, f = y ~ x1 + x2 + x3 | g,
+                         s2 = "phase2",
+                         auxiliaryWeights = "weights")
> (out <- maSAE::predict(object, use_lm = FALSE))
```

```
  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   406.4824 494.2358 394.8737   304.2430 406.3514   534.8764
2         b   430.0021 413.9014 423.3800   317.5597 430.0255   458.3616

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> ###% three-phase
> object <- maSAE::saObj(data = s012, f = y ~ x1 + x2 + x3 | g,
+                        s1 = "phase1", s2 = "phase2",
+                        auxiliaryWeights = "weights")
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   424.8187 322.9752 413.1992   86.27596 424.6769   316.9094
2         b   437.5184 272.6481 430.9232   86.47232 437.5686   227.2741

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> #% clustered
> ##% un-weighted
> ###% two-phase
> ####% partially exhaustive
> object <- maSAE::saObj(data = s12, f = y ~ x1 + x2 + x3 | g, s2 = "phase2", smallAreaMea
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   377.0824 549.1471 363.3825   100.3859 376.8559   556.3546
2         b   385.1621 458.4486 380.3530   107.9309 385.0648   396.8564

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> ####% exhaustive
> object <- maSAE::saObj(data = s12, f = y ~ x1 + x2 + x3 | g, s2 = "phase2", smallAreaMea
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   368.1216 428.7513 354.4524   74.40279 367.9259   530.3716
2         b   381.1313 328.3773 376.3492   84.26918 381.0611   373.1947

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE
```

```
> ####% non-exhaustive
> object <- maSAE::saObj(data = s12, f = y ~ x1 + x2 + x3 | g, s2 = "phase2", cluster = "c
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance  psynth var_psynth   psmall var_psmall
1         a   381.5729 950.4594 367.868   580.6438 381.3414  1036.6126
2         b   392.3395 807.3224 387.575   575.5981 392.2869   864.5236

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> ###% three-phase
> object <- maSAE::saObj(data = s012, f = y ~ x1 + x2 + x3 | g, s1 = "phase1", s2 = "phase
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   400.3065 600.9477 386.5807   135.3258 400.0542   591.2946
2         b   404.9677 493.6005 400.2816   143.9249 404.9935   432.8504

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> ##% weighted
> ###% two-phase
> ####% partially exhaustive
> object <- maSAE::saObj(data = s12, f = y ~ x1 + x2 + x3 | g, s2 = "phase2", smallAreaMea
+                         auxiliaryWeights = "weights")
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   376.6663 550.5192 363.4477   100.4981 376.4431   558.1007
2         b   384.8009 457.7284 380.2073   107.8563 384.7049   396.4425

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> ####% exhaustive
> object <- maSAE::saObj(data = s12, f = y ~ x1 + x2 + x3 | g, s2 = "phase2", smallAreaMea
+                         auxiliaryWeights = "weights")
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   368.2336 430.3318 355.0431   74.32785 368.0385   531.9304
2         b   381.3361 327.0091 376.7705   83.95160 381.2681   372.5378

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)
```

```
[1] TRUE

> ####% non-exhaustive
> object <- maSAE::saObj(data = s12, f = y ~ x1 + x2 + x3 | g, s2 = "phase2", cluster = "c
+                        auxiliaryWeights = "weights")
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   381.1568 950.9773 367.9332   579.8245 380.9286  1037.4271
2         b   391.9784 805.4768 387.4293   574.4109 391.9270   862.9971

> outlm <- maSAE::predict(object, use_lm = TRUE)
> RUnit::checkEquals(out, outlm)

[1] TRUE

> ###% three-phase
> object <- maSAE::saObj(data = s012, f = y ~ x1 + x2 + x3 | g, s1 = "phase1", s2 = "phase
+                        auxiliaryWeights = "weights")
> (out <- maSAE::predict(object, use_lm = FALSE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   399.8904 602.3198 386.6459   135.4381 399.6414   593.0406
2         b   404.6066 492.8803 400.1359   143.8503 404.6336   432.4365

> (outlm <- maSAE::predict(object, use_lm = TRUE))

  smallArea prediction variance   psynth var_psynth   psmall var_psmall
1         a   399.8904 602.3198 386.6459   135.4381 399.6414   593.0406
2         b   404.6066 492.8803 400.1359   143.8503 404.6336   432.4365

> RUnit::checkEquals(out, outlm)

[1] TRUE

>
>
```

References

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