Package ‘tessellation’

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Type Package

Title Delaunay and Voronoï Tessellations

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Maintainer Stéphane Laurent <laurent_step@outlook.fr>

Description Delaunay and Voronoï tessellations, with emphasis on the two-dimensional and the three-dimensional cases (the package provides functions to plot the tessellations for these cases). Delaunay tessellations are computed in C with the help of the 'Qhull' library <http://www.qhull.org/>.

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URL https://github.com/stla/tessellation, https://stla.github.io/tessellation/

BugReports https://github.com/stla/tessellation/issues

Imports colorsGen, cxhull, english, graphics, hash, Polychrome, R6, rgl, Rvcg, scales, sets, utils

Suggests knitr, paletteer, rmarkdown, uniformly, viridisLite

Encoding UTF-8

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NeedsCompilation yes

Author Stéphane Laurent [aut, cre], C. B. Barber [cph] (author of the Qhull library), The Geometry Center [cph]

Repository CRAN

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R topics documented:

`cellVertices` *Vertices of a bounded cell*

Description

Get all vertices of a bounded cell, without duplicates.

Usage

```
cellVertices(cell, check.bounded = TRUE)
```

Arguments

`cell`	a bounded Voronoï cell
`check.bounded`	Boolean, whether to check that the cell is bounded; set to `FALSE` for a small speed gain if you know that the cell is bounded

Value

A matrix, each row represents a vertex.

Examples

```
library(tessellation)
d <- delaunay(centricCuboctahedron())
v <- voronoi(d)
cell13 <- v[[13]]
isBoundedCell(cell13) # TRUE
library(rgl)
open3d(windowRect = c(50, 50, 562, 562))
invisible(lapply(cell13[["cell"]], function(edge){
  edge$plot(edgeAsTube = TRUE, tubeRadius = 0.025, tubeColor = "yellow")
}))
cellvertices <- cellVertices(cell13)
spheres3d(cellvertices, radius = 0.1, color = "green")
```

---

cellVolume *Volume of a bounded Voronoï cell*

Description

For a bounded 2D Voronoï cell, returns the area of the cell, and for a bounded 3D Voronoï cell, returns the volume of the cell and its surface area.

Usage

```
cellVolume(cell)
```

Arguments

`cell`	a bounded 2D or 3D Voronoï cell

Value

A number, the area/volume of the cell, and in the 3D case, the surface area of the cell is attached to this number as an attribute.

Examples

```
library(tessellation)
d <- delaunay(centricCuboctahedron())
v <- voronoi(d)
cell13 <- v[[13]]
isBoundedCell(cell13) # TRUE
cellVolume(cell13)
```

`centricCuboctahedron` *Centric cuboctahedron*

Description

A cuboctahedron (12 vertices), with a point added at its center.

Usage

```
centricCuboctahedron()
```

Value

A numeric matrix with 13 rows and 3 columns.

`delaunay` *Delaunay triangulation*

Description

Delaunay triangulation (or tessellation) of a set of points.

Usage

```
delaunay(
  points,
  atinfinity = FALSE,
  degenerate = FALSE,
  exteriorEdges = FALSE,
  elevation = FALSE
)
```

Arguments

`points`	the points given as a matrix, one point per row
`atinfinity`	Boolean, whether to include a point at infinity; ignored if `elevation=TRUE`
`degenerate`	Boolean, whether to include degenerate tiles; ignored if `elevation=TRUE`
`exteriorEdges`	Boolean, for dimension 3 only, whether to return the exterior edges (see below)
`elevation`	Boolean, only for three-dimensional points; if `TRUE`, the function performs an elevated Delaunay triangulation (also called 2.5D Delaunay triangulation), using the third coordinate of a point as its elevation; see the example

Value

If the function performs an elevated Delaunay tessellation, then the returned value is a list with four fields: `mesh`, `edges`, `volume`, and `surface`. The `mesh` field is an object of class `mesh3d`, ready for plotting with the **rgl** package. The `edges` field is an integer matrix which provides the indices of the vertices of the edges, and an indicator of whether an edge is a border edge; this matrix is obtained with `vcgGetEdge`. The `volume` field provides the sum of the volumes under the Delaunay triangles, that is to say the total volume under the triangulated surface. Finally, the `surface` field provides the sum of the areas of the Delaunay triangles, thus this is an approximate value of the area of the surface that is triangulated. The elevated Delaunay tessellation is built with the help of the **interp** package.

Otherwise, the function returns the Delaunay tessellation with many details, in a list. This list contains five fields:

vertices the vertices (or sites) of the tessellation; these are the points passed to the function

tiles the tiles of the tessellation (triangles in dimension 2, tetrahedra in dimension 3)

tilefacets the facets of the tiles of the tessellation

mesh a 'rgl' mesh (`mesh3d` object)

edges a two-columns integer matrix representing the edges, each row represents an edge; the two integers of a row are the indices of the two points which form the edge.

In dimension 3, the list contains an additional field *exteriorEdges* if you set `exteriorEdges = TRUE`. This is the list of the exterior edges, represented as `Edge3` objects. This field is involved in the function `plotDelaunay3D`.

The **vertices** field is a list with the following fields:

id the id of the vertex; this is nothing but the index of the corresponding point passed to the function

neighvertices the ids of the vertices of the tessellation connected to this vertex by an edge

neightilefacets the ids of the tile facets this vertex belongs to

neightiles the ids of the tiles this vertex belongs to

The **tiles** field is a list with the following fields:

id the id of the tile

simplex a list describing the simplex (that is, the tile); this list contains four fields: *vertices*, a `hash` giving the simplex vertices and their id, *circumcenter*, the circumcenter of the simplex, *circumradius*, the circumradius of the simplex, and *volume*, the volume of the simplex

facets the ids of the facets of this tile

neighbors the ids of the tiles adjacent to this tile

family two tiles have the same family if they share the same circumcenter; in this case the family is an integer, and the family is `NA` for tiles which do not share their circumcenter with any other tile

orientation `1` or `-1`, an indicator of the orientation of the tile

The **tilefacets** field is a list with the following fields:

id the id of this tile facet

subsimplex a list describing the subsimplex (that is, the tile facet); this list is similar to the *simplex* list of **tiles**

facetOf one or two ids, the id(s) of the tile this facet belongs to

normal a vector, the normal of the tile facet

offset a number, the offset of the tile facet

Note

The package provides the functions `plotDelaunay2D` to plot a 2D Delaunay tessellation and `plotDelaunay3D` to plot a 3D Delaunay tessellation. But there is no function to plot an elevated Delaunay tessellation; the examples show how to plot such a Delaunay tessellation.

See Also

`getDelaunaySimplices`

Examples

```
library(tessellation)
points <- rbind(
 c(0.5,0.5,0.5),
 c(0,0,0),
 c(0,0,1),
 c(0,1,0),
 c(0,1,1),
 c(1,0,0),
 c(1,0,1),
 c(1,1,0),
 c(1,1,1)
)
del <- delaunay(points)
del$vertices[[1]]
del$tiles[[1]]
del$tilefacets[[1]]

# an elevated Delaunay tessellation ####
f <- function(x, y){
  dnorm(x) * dnorm(y)
}
x <- y <- seq(-5, 5, length.out = 50)
grd <- expand.grid(x = x, y = y) # grid on the xy-plane
points <- as.matrix(transform( # data (x_i, y_i, z_i)
  grd, z = f(x, y)
))
del <- delaunay(points, elevation = TRUE)
del[["volume"]] # close to 1, as expected
# plotting
library(rgl)
mesh <- del[["mesh"]]
open3d(windowRect = c(100, 100, 612, 356), zoom = 0.6)
aspect3d(1, 1, 20)
```

```
shade3d(mesh, color = "limegreen", polygon_offset = 1)
wire3d(mesh)

# another elevated Delaunay triangulation, to check the correctness
#   of the calculated surface and the calculated volume ####
library(Rvcg)
library(rgl)
cap <- vcgSphericalCap(angleRad = pi/2, subdivision = 3)
open3d(windowRect = c(100, 100, 612, 356), zoom = 0.6)
shade3d(cap, color = "lawngreen", polygon_offset = 1)
wire3d(cap)
# exact value of the surface of the spherical cap:
R <- 1
h <- R * (1 - sin(pi/2/2))
2 * pi * R * h
# our approximation:
points <- t(cap$vb[-4, ]) # the points on the spherical cap
del <- delaunay(points, elevation = TRUE)
del[["surface"]]
# try to increase `subdivision` in `vcgSphericalCap` to get a
#   better approximation of the true value
# note that 'Rvcg' returns the same result as ours:
vcgArea(cap)
# let's check the volume as well:
pi * h^2 * (R - h/3) # true value
del[["volume"]]
# there's a warning with 'Rvcg':
tryCatch(vcgVolume(cap), warning = function(w) message(w))
suppressWarnings({vcgVolume(cap)})
```

Edge2 *R6 class representing an edge in dimension 2.*

Description

An edge is given by two vertices in the 2D space, named A and B. This is for example an edge of a Voronoï cell of a 2D Delaunay tessellation.

Active bindings

A get or set the vertex A

B get or set the vertex B

Methods

Public methods:

- Edge2$new()
- Edge2$print()
- Edge2$plot()

- Edge2$stack()
- Edge2$clone()

Method `new()`**:** Create a new `Edge2` object.

Usage:

```
Edge2$new(A, B)
```

Arguments:

`A` the vertex `A`

`B` the vertex `B`

Returns: A new `Edge2` object.

Examples:

```
edge <- Edge2$new(c(1, 1), c(2, 3))
edge
edge$A
edge$A <- c(1, 0)
edge
```

Method `print()`**:** Show instance of an `Edge2` object.

Usage:

```
Edge2$print(...)
```

Arguments:

`...` ignored

Examples:

```
Edge2$new(c(2, 0), c(3, -1))
```

Method `plot()`**:** Plot an `Edge2` object.

Usage:

```
Edge2$plot(color = "black", ...)
```

Arguments:

`color` the color of the edge

`...` graphical parameters such as `lty` or `lwd`

Examples:

```
library(tessellation)
centricSquare <- rbind(
  c(-1, 1), c(1, 1), c(1, -1), c(-1, -1), c(0, 0)
)
d <- delaunay(centricSquare)
v <- voronoi(d)
cell5 <- v[[5]] # the cell of the point (0, 0), at the center
isBoundedCell(cell5) # TRUE
plot(centricSquare, type = "n")
invisible(lapply(cell5[["cell"]], function(edge) edge$plot()))
```

Method `stack()`**:** Stack the two vertices of the edge (this is for internal purpose).

Usage:

`Edge2$stack()`

Method `clone()`**:** The objects of this class are cloneable with this method.

Usage:

`Edge2$clone(deep = FALSE)`

Arguments:

`deep` Whether to make a deep clone.

Examples

```
## ------------------------------------------------
## Method `Edge2$new`
## ------------------------------------------------

edge <- Edge2$new(c(1, 1), c(2, 3))
edge
edge$A
edge$A <- c(1, 0)
edge

## ------------------------------------------------
## Method `Edge2$print`
## ------------------------------------------------

Edge2$new(c(2, 0), c(3, -1))

## ------------------------------------------------
## Method `Edge2$plot`
## ------------------------------------------------

library(tessellation)
centricSquare <- rbind(
  c(-1, 1), c(1, 1), c(1, -1), c(-1, -1), c(0, 0)
)
d <- delaunay(centricSquare)
v <- voronoi(d)
cell5 <- v[[5]] # the cell of the point (0, 0), at the center
isBoundedCell(cell5) # TRUE
plot(centricSquare, type = "n")
invisible(lapply(cell5[["cell"]], function(edge) edge$plot()))
```

`Edge3` *R6 class representing an edge in dimension 3.*

Description

An edge is given by two vertices in the 3D space, named `A` and `B`. This is for example an edge of a Voronoï cell of a 3D Delaunay tessellation.

Active bindings

`A` get or set the vertex `A`

`B` get or set the vertex `B`

`idA` get or set the id of vertex `A`

`idB` get or set the id of vertex `B`

Methods

Public methods:

- `Edge3$new()`
- `Edge3$print()`
- `Edge3$plot()`
- `Edge3$stack()`
- `Edge3$clone()`

Method `new()`**:** Create a new `Edge3` object.

Usage:

```
Edge3$new(A, B, idA, idB)
```

Arguments:

`A` the vertex `A`

`B` the vertex `B`

`idA` the id of vertex `A`, an integer; can be missing

`idB` the id of vertex `B`, an integer; can be missing

Returns: A new `Edge3` object.

Examples:

```
edge <- Edge3$new(c(1, 1, 1), c(1, 2, 3))
edge
edge$A
edge$A <- c(1, 0, 0)
edge
```

Method `print()`**:** Show instance of an `Edge3` object.

Usage:

```
Edge3$print(...)
```

Arguments:

`...` ignored

Examples:

```
Edge3$new(c(2, 0, 0), c(3, -1, 4))
```

Method `plot()`**:** Plot an `Edge3` object.

Usage:

```
Edge3$plot(edgeAsTube = FALSE, tubeRadius, tubeColor)
```

Arguments:

`edgeAsTube` Boolean, whether to plot the edge as a tube

`tubeRadius` the radius of the tube

`tubeColor` the color of the tube

Examples:

```
library(tessellation)
d <- delaunay(centricCuboctahedron())
v <- voronoi(d)
cell13 <- v[[13]] # the point (0, 0, 0), at the center
isBoundedCell(cell13) # TRUE
library(rgl)
open3d(windowRect = c(50, 50, 562, 562))
invisible(lapply(cell13[["cell"]], function(edge) edge$plot()))
```

Method `stack()`**:** Stack the two vertices of the edge (this is for internal purpose).

Usage:

```
Edge3$stack()
```

Method `clone()`**:** The objects of this class are cloneable with this method.

Usage:

```
Edge3$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

Examples

```
## ------------------------------------------------
## Method `Edge3$new`
## ------------------------------------------------

edge <- Edge3$new(c(1, 1, 1), c(1, 2, 3))
edge
edge$A
edge$A <- c(1, 0, 0)
edge

## ------------------------------------------------
## Method `Edge3$print`
## ------------------------------------------------

Edge3$new(c(2, 0, 0), c(3, -1, 4))

## ------------------------------------------------
## Method `Edge3$plot`
## ------------------------------------------------

library(tessellation)
```

```
d <- delaunay(centricCuboctahedron())
v <- voronoi(d)
cell13 <- v[[13]] # the point (0, 0, 0), at the center
isBoundedCell(cell13) # TRUE
library(rgl)
open3d(windowRect = c(50, 50, 562, 562))
invisible(lapply(cell13[["cell"]], function(edge) edge$plot()))
```

`getDelaunaySimplices` *Delaunay simplices*

Description

Get Delaunay simplices (tiles).

Usage

```
getDelaunaySimplices(tessellation, hashes = FALSE)

getDelaunaySimplicies(tessellation, hashes = FALSE)
```

Arguments

`tessellation`	the output of `delaunay`
`hashes`	Boolean, whether to return the simplices as hash maps

Value

The list of simplices of the Delaunay tessellation.

Examples

```
library(tessellation)
pts <- rbind(
  c(-5, -5,  16),
  c(-5,  8,   3),
  c(4,  -1,   3),
  c(4,  -5,   7),
  c(4,  -1, -10),
  c(4,  -5, -10),
  c(-5,  8, -10),
  c(-5, -5, -10)
)
tess <- delaunay(pts)
getDelaunaySimplices(tess)
```

IEdge2 *R6 class representing a semi-infinite edge in dimension 2*

Description

A semi-infinite edge is given by a vertex, its origin, and a vector, its direction. Voronoï diagrams possibly have such edges.

Active bindings

`O` get or set the vertex `O`

`direction` get or set the vector `direction`

Methods

Public methods:

- `IEdge2$new()`
- `IEdge2$print()`
- `IEdge2$clone()`

Method `new()`**:** Create a new `IEdge2` object.

Usage:

```
IEdge2$new(O, direction)
```

Arguments:

`O` the vertex `O` (origin)

`direction` the vector `direction`

Returns: A new `IEdge2` object.

Examples:

```
iedge <- IEdge2$new(c(1, 1), c(2, 3))
iedge
iedge$O
iedge$O <- c(1, 0)
iedge
```

Method `print()`**:** Show instance of an `IEdge2` object.

Usage:

```
IEdge2$print(...)
```

Arguments:

`...` ignored

Examples:

```
IEdge2$new(c(2, 0), c(3, -1))
```

Method `clone()`**:** The objects of this class are cloneable with this method.

Usage:

```
IEdge2$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

Examples

```
## ------------------------------------------------
## Method `IEdge2$new`
## ------------------------------------------------

iedge <- IEdge2$new(c(1, 1), c(2, 3))
iedge
iedge$O
iedge$O <- c(1, 0)
iedge

## ------------------------------------------------
## Method `IEdge2$print`
## ------------------------------------------------

IEdge2$new(c(2, 0), c(3, -1))
```

`IEdge3` *R6 class representing a semi-infinite edge in dimension 3*

Description

A semi-infinite edge is given by a vertex, its origin, and a vector, its direction. Voronoï diagrams possibly have such edges.

Active bindings

`O` get or set the vertex `O`

`direction` get or set the vector `direction`

Methods

Public methods:

- `IEdge3$new()`
- `IEdge3$print()`
- `IEdge3$clone()`

Method `new()`**:** Create a new `IEdge3` object.

Usage:

```
IEdge3$new(O, direction)
```

Arguments:

`O` the vertex `O` (origin)

`direction` the vector `direction`

Returns: A new `IEdge3` object.

Examples:

```
iedge <- IEdge3$new(c(1, 1, 1), c(1, 2, 3))
iedge
iedge$O
iedge$O <- c(1, 0, 0)
iedge
```

Method `print()`**:** Show instance of an `IEdge3` object.

Usage:

```
IEdge3$print(...)
```

Arguments:

`...` ignored

Examples:

```
IEdge3$new(c(2, 0, 0), c(3, -1, 4))
```

Method `clone()`**:** The objects of this class are cloneable with this method.

Usage:

```
IEdge3$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

Examples

```
## ------------------------------------------------
## Method `IEdge3$new`
## ------------------------------------------------

iedge <- IEdge3$new(c(1, 1, 1), c(1, 2, 3))
iedge
iedge$O
iedge$O <- c(1, 0, 0)
iedge

## ------------------------------------------------
## Method `IEdge3$print`
## ------------------------------------------------

IEdge3$new(c(2, 0, 0), c(3, -1, 4))
```

`isBoundedCell` *Is this cell bounded?*

Description

Check whether a Voronoï cell is bounded, i.e. contains only finite edges.

Usage

```
isBoundedCell(cell)
```

Arguments

`cell`	a Voronoï cell

Value

A Boolean value, whether the cell is bounded.

`plotBoundedCell2D` *Plot a bounded Voronoï 2D cell*

Description

Plot a bounded Voronoï 2D cell.

Usage

```
plotBoundedCell2D(
  cell,
  border = "black",
  color = NA,
  check.bounded = TRUE,
  ...
)
```

Arguments

`cell`	a bounded Voronoï 2D cell
`border`	color of the borders of the cell; `NA` for no color
`color`	color of the cell; `NA` for no color
`check.bounded`	Boolean, whether to check that the cell is bounded; set to `FALSE` for a small speed gain if you know that the cell is bounded
`...`	graphical parameters for the borders

Value

No value, this function just plots the cell (more precisely, it adds the plot of the cell to the current plot).

Examples

```
library(tessellation)
centricSquare <- rbind(
  c(-1, 1), c(1, 1), c(1, -1), c(-1, -1), c(0, 0)
)
d <- delaunay(centricSquare)
v <- voronoi(d)
cell5 <- v[[5]]
isBoundedCell(cell5) # TRUE
plot(centricSquare, type = "n", asp = 1, xlab = "x", ylab = "y")
plotBoundedCell2D(cell5, color = "pink")
```

`plotBoundedCell3D` *Plot a bounded Voronoï 3D cell*

Description

Plot a bounded Voronoï 3D cell with **rgl**.

Usage

```
plotBoundedCell3D(
  cell,
  edgesAsTubes = FALSE,
  tubeRadius,
  tubeColor,
  facetsColor = NA,
  alpha = 1,
  check.bounded = TRUE
)
```

Arguments

`cell`	a bounded Voronoï 3D cell
`edgesAsTubes`	Boolean, whether to plot edges as tubes or as lines
`tubeRadius`	radius of the tubes if `edgesAsTubes = TRUE`
`tubeColor`	color of the tubes if `edgesAsTubes = TRUE`
`facetsColor`	color of the facets; `NA` for no color
`alpha`	opacity of the facets, a number between 0 and 1
`check.bounded`	Boolean, whether to check that the cell is bounded; set to `FALSE` for a small speed gain if you know that the cell is bounded

Value

No value, this function just plots the cell.

Examples

```
library(tessellation)
d <- delaunay(centricCuboctahedron())
v <- voronoi(d)
cell13 <- v[[13]]
isBoundedCell(cell13) # TRUE
library(rgl)
open3d(windowRect = c(50, 50, 562, 562))
plotBoundedCell3D(
  cell13, edgesAsTubes = TRUE, tubeRadius = 0.03, tubeColor = "yellow",
  facetsColor = "navy", alpha = 0.7
)
```

`plotDelaunay2D` *Plot 2D Delaunay tessellation*

Description

Plot a 2D Delaunay tessellation.

Usage

```
plotDelaunay2D(
  tessellation,
  border = "black",
  color = "distinct",
  distinctArgs = list(seedcolors = c("#ff0000", "#00ff00", "#0000ff")),
  randomArgs = list(hue = "random", luminosity = "bright"),
  lty = par("lty"),
  lwd = par("lwd"),
  ...
)
```

Arguments

`tessellation`	the output of `delaunay`
`border`	the color of the borders of the triangles; `NULL` for no borders
`color`	controls the filling colors of the triangles, either `FALSE` for no color, `"random"` to use `randomColor`, or `"distinct"` to use `createPalette`
`distinctArgs`	if `color = "distinct"`, a list of arguments passed to `createPalette`
`randomArgs`	if `color = "random"`, a list of arguments passed to `randomColor`
`lty, lwd`	graphical parameters
`...`	arguments passed to `plot`

Value

No value, just renders a 2D plot.

Examples

```
# random points in a square
set.seed(314)
library(tessellation)
library(uniformly)
square <- rbind(
  c(-1, 1), c(1, 1), c(1, -1), c(-1, -1)
)
ptsin <- runif_in_cube(10L, d = 2L)
pts <- rbind(square, ptsin)
d <- delaunay(pts)
opar <- par(mar = c(0, 0, 0, 0))
plotDelaunay2D(
  d, xlab = NA, ylab = NA, asp = 1, color = "random",
  randomArgs = list(hue = "random", luminosity = "dark")
)
par(opar)
```

---

plotDelaunay3D — *Plot 3D Delaunay tessellation*

Description

Plot a 3D Delaunay tessellation with **rgl**.

Usage

```
plotDelaunay3D(
  tessellation,
  color = "distinct",
  distinctArgs = list(seedcolors = c("#ff0000", "#00ff00", "#0000ff")),
  randomArgs = list(hue = "random", luminosity = "bright"),
  alpha = 0.3,
  exteriorEdgesAsTubes = FALSE,
  tubeRadius,
  tubeColor
)
```

Arguments

`tessellation`	the output of `delaunay`
`color`	controls the filling colors of the tetrahedra, either `FALSE` for no color, `"random"` to use `randomColor`, or `"distinct"` to use `createPalette`
`distinctArgs`	if color = `"distinct"`, a list of arguments passed to `createPalette`

`randomArgs`	if `color = "random"`, a list of arguments passed to `randomColor`
`alpha`	opacity, number between 0 and 1
`exteriorEdgesAsTubes`	Boolean, whether to plot the exterior edges as tubes; in order to use this feature, you need to set `exteriorEdges = TRUE` in the `delaunay` function
`tubeRadius`	if `exteriorEdgesAsTubes = TRUE`, the radius of the tubes
`tubeColor`	if `exteriorEdgesAsTubes = TRUE`, the color of the tubes

Value

No value, just renders a 3D plot.

Examples

```
library(tessellation)
pts <- rbind(
  c(-5, -5,  16),
  c(-5,  8,   3),
  c(4,  -1,   3),
  c(4,  -5,   7),
  c(4,  -1, -10),
  c(4,  -5, -10),
  c(-5,  8, -10),
  c(-5, -5, -10)
)
tess <- delaunay(pts)
library(rgl)
open3d(windowRect = c(50, 50, 562, 562))
plotDelaunay3D(tess, color = "random")
open3d(windowRect = c(50, 50, 562, 562))
plotDelaunay3D(
  tess, exteriorEdgesAsTubes = TRUE, tubeRadius = 0.3, tubeColor = "yellow"
)
```

`plotVoronoiDiagram` *Plot Voronoï diagram*

Description

Plot all the bounded cells of a 2D or 3D Voronoï tessellation.

Usage

```
plotVoronoiDiagram(
  v,
  colors = "random",
  distinctArgs = list(seedcolors = c("#ff0000", "#00ff00", "#0000ff")),
  randomArgs = list(hue = "random", luminosity = "bright"),
```

```
  alpha = 1,
  ...
)
```

Arguments

v	an output of voronoi
colors	this can be "random" to use random colors for the cells with randomColor, "distinct" to use distinct colors with the help of createPalette, or this can be NA for no colors, or a vector of colors; the length of this vector of colors must match the number of bounded cells, which is displayed when you run the voronoi function and that you can also get by typing attr(v, "nbounded")
distinctArgs	if colors = "distinct", a list of arguments passed to createPalette
randomArgs	if colors = "random", a list of arguments passed to randomColor
alpha	opacity, a number between 0 and 1 (used when colors is not NA)
...	arguments passed to plotBoundedCell2D or plotBoundedCell3D

Value

No returned value.

Note

Sometimes, it is necessary to set the option degenerate=TRUE in the delaunay function in order to get a correct Voronoï diagram with the plotVoronoiDiagram function (I don't know why).

Examples

```
library(tessellation)
# 2D example: Fermat spiral
theta <- seq(0, 100, length.out = 300L)
x <- sqrt(theta) * cos(theta)
y <- sqrt(theta) * sin(theta)
pts <- cbind(x,y)
opar <- par(mar = c(0, 0, 0, 0), bg = "black")
# Here is a Fermat spiral:
plot(pts, asp = 1, xlab = NA, ylab = NA, axes = FALSE, pch = 19, col = "white")
# And here is its Voronoï diagram:
plot(NULL, asp = 1, xlim = c(-15, 15), ylim = c(-15, 15),
     xlab = NA, ylab = NA, axes = FALSE)
del <- delaunay(pts)
v <- voronoi(del)
length(Filter(isBoundedCell, v)) # 281 bounded cells
plotVoronoiDiagram(v, colors = viridisLite::turbo(281L))
par(opar)

# 3D example: tetrahedron surrounded by three circles
tetrahedron <-
  rbind(
    c(2*sqrt(2)/3, 0, -1/3),
```

```
    c(-sqrt(2)/3, sqrt(2/3), -1/3),
    c(-sqrt(2)/3, -sqrt(2/3), -1/3),
    c(0, 0, 1)
  )
angles <- seq(0, 2*pi, length.out = 91)[-1]
R <- 2.5
circle1 <- t(vapply(angles, function(a) R*c(cos(a), sin(a), 0), numeric(3L)))
circle2 <- t(vapply(angles, function(a) R*c(cos(a), 0, sin(a)), numeric(3L)))
circle3 <- t(vapply(angles, function(a) R*c(0, cos(a), sin(a)), numeric(3L)))
circles <- rbind(circle1, circle2, circle3)
pts <- rbind(tetrahedron, circles)
d <- delaunay(pts, degenerate = TRUE)
v <- voronoi(d)
library(rgl)
open3d(windowRect = c(50, 50, 562, 562))
material3d(lwd = 2)
plotVoronoiDiagram(v)
```

`surface` *Tessellation surface*

Description

Exterior surface of the Delaunay tessellation.

Usage

```
surface(tessellation)
```

Arguments

`tessellation`	output of `delaunay`

Value

A number, the exterior surface of the Delaunay tessellation (perimeter in 2D).

Note

It is not guaranteed that this function provides the correct result for all cases. The exterior surface of the Delaunay tessellation is the exterior surface of the convex hull of the sites (the points), and you can get it with the **cxhull** package (by summing the volumes of the facets). Moreover, I encountered some cases for which I got a correct result only with the option `degenerate=TRUE` in the `delaunay` function. I will probably remove this function in the next version.

See Also

`volume`

teapot — *Utah teapot*

Description

Vertices of the Utah teapot.

Usage

```
teapot()
```

Value

A matrix with 1976 rows and 3 columns.

tessellation-imports — *Objects imported from other packages*

Description

These objects are imported from other packages. Follow the links to their documentation: `values`, `keys`.

volume — *Tessellation volume*

Description

The volume of the Delaunay tessellation, that is, the volume of the convex hull of the sites.

Usage

```
volume(tessellation)
```

Arguments

`tessellation`	output of `delaunay`

Value

A number, the volume of the Delaunay tessellation (area in 2D).

See Also

`surface`

voronoi *Voronoï tessellation*

Description

Voronoï tessellation from Delaunay tessellation; this is a list of pairs made of a site (a vertex) and a list of edges.

Usage

```
voronoi(tessellation)
```

Arguments

tessellation output of `delaunay`

Value

A list of pairs representing the Voronoï tessellation. Each `pair` is named: the first component is called `"site"`, and the second component is called `"cell"`.

See Also

`isBoundedCell`, `cellVertices`, `plotBoundedCell2D`, `plotBoundedCell3D`

Examples

```
library(tessellation)
d <- delaunay(centricCuboctahedron())
v <- voronoi(d)
# the Voronoï diagram has 13 cells (one for each site):
length(v)
# there is only one bounded cell:
length(Filter(isBoundedCell, v)) # or attr(v, "nbounded")
```

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