

# Demand Analysis with the “Almost Ideal Demand System” in R: Package `micEconAids`

Arne Henningsen

Department of Food and Resource Economics, University of Copenhagen

---

## Abstract

The R package `micEconAids` provides a convenient interface for demand analysis with the Almost Ideal Demand System (AIDS) and its linear approximation (LA-AIDS). The user can choose between various approaches for the econometric estimation, e.g. the “Iterative Linear Least-Squares Estimator.” Furthermore, many different formulas for calculating demand elasticities are available. A special feature is the calculation of the variance-covariance matrix and the standard errors of the demand elasticities of the AIDS.

*Keywords:* demand analysis, almost ideal demand system, demand elasticities, R.

---

## 1. Introduction

The “Almost Ideal Demand System” (AIDS) proposed by [Deaton and Muellbauer \(1980a,b\)](#) is probably the most popular demand system in empirical demand analysis. It has been applied not only for many studies of general household demand, but also in various specific fields such as analyzing labor/leisure choice, financing decisions, sensitivity to price changes of private labels, patent licensing values, or greenhouse gas emissions. Although the AIDS model is more than 25 years old, this functional form is still widely used in recent publications (see, e.g., [Brännlund, Ghalwash, and Nordström 2007](#); [Chambwera and Folmer 2007](#); [Farrell and Shields 2007](#); [Hausman and Leonard 2007](#); [Henning and Henningsen 2007](#); [Huang, Jones, and Hahn 2007](#); [Moore and Green 2007](#); [Raknerud, Skjerpen, and Swensen 2007](#); [West and Williams 2007](#); [Xiao, Zarnikau, and Damien 2007](#)), because it unifies almost all theoretically and empirically desirable properties. However, several scholars have shown that the econometric estimation is paved with a number of pitfalls. For instance, the linear approximation of the AIDS (LA-AIDS) using the Stone price index, which is applied in most empirical studies, leads to severe econometric and theoretical problems. Therefore, a vast literature evolved proposing several different approaches that try to circumvent these problems.

The aim of this paper is to describe the tools for demand analysis with the AIDS that are available in the R ([R Development Core Team 2008](#)) package `micEconAids` (version 0.5). Among several other tools for microeconomic analysis, this package provides the function `aidsEst` which is a convenient user interface for the econometric estimation of an AIDS. The user can choose between various estimation approaches such as the linear approximate AIDS (LA-AIDS) with several different price indices and the Iterative Linear Least Squares

Estimator (ILLE). Furthermore, the function `aidsEla` and the method `elas` can calculate demand elasticities and their standard errors by several different formulas that can be found in the literature.

The tools for demand analysis with the AIDS that are available in the **micEconAids** package have several advantages compared to other implementations of the AIDS model (e.g., Poi 2002; SAS 2008b,a): (i) the **micEconAids** package entirely relies on free software so that it can be tested and used instantly without any license restrictions by everyone (e.g., by students of a course in microeconomic modeling); (ii) the **micEconAids** package itself as well as all software packages utilized by **micEconAids** are open-source software so that users can (peer) review the algorithms and — if necessary — modify or extend the source code to meet specific requirements; (iii) the **micEconAids** package includes many different approaches for demand analysis with the AIDS that are generally not available in other implementations; (iv) the tools in the **micEconAids** package provide various user options so that users can easily switch between these different approaches; (v) the tools in the **micEconAids** package are much user-friendlier than other implementations so that even users with virtually no programming skills can perform a demand analysis with the AIDS (e.g., the estimation of the AIDS can be done by a single command). However, there are also a few features of other implementations that are currently not available in the **micEconAids** package; it is planned to add these missing features in the future (see Section 7).

The following section of this paper describes the AIDS and LA-AIDS and how these models can be estimated with the **micEconAids** package. The third section discusses consistency with economic theory and explains how this can be imposed or checked. The calculation of demand elasticities is described in the fourth section. The prediction of expenditure shares and demanded quantities is presented in the fifth section. The sixth section uses the **micEconAids** package to replicate two demand analyses that are published in the literature. The last section summarizes and concludes.

## 2. Estimating the AIDS

The “Almost Ideal Demand System” (AIDS) is widely used in applied demand analysis, because it has many desirable properties: (a) it is an arbitrary first-order approximation to any demand system; (b) it satisfies the axioms of choice; (c) it aggregates exactly over consumers while still allowing non-linear Engel curves, (d) the homogeneity and symmetry property can be tested and imposed by simple parameter restrictions and (e) the demand equations become linear if the translog price index is approximated e.g. by the Stone index (Deaton and Muellbauer 1980a, p. 312). However, it seems that Deaton and Muellbauer were a little bit too optimistic — at least regarding the last property. We will come back to this issue later.

The AIDS is based on microeconomic household theory. The starting point of the model is the expenditure function<sup>1</sup>  $e(p_t, U_t)$  with

$$\ln m_t = \ln e(p_t, U_t) = \alpha_0 + \sum_i \alpha_i \ln p_{it} + \frac{1}{2} \sum_i \sum_j \gamma_{ij}^* \ln p_{it} \ln p_{jt} + U_t \beta_0 \prod_i p_{it}^{\beta_i}, \quad (1)$$

where  $m_t$  is total expenditure at time  $t$ ,  $p_{it}$  is the price of good  $i$  at time  $t$ ,  $U_t$  is the utility

<sup>1</sup>The economic terms that are used in this paper (e.g., “expenditure function”) are explained Appendix A.

level at time  $t$ , and  $\alpha$ ,  $\beta$  and  $\gamma^*$  are coefficients.

Marshallian (uncompensated) demand functions can be derived from the expenditure function (1) by applying Shephard's Lemma and replacing the unobservable utility level by the indirect utility function that corresponds to the expenditure function (Deaton and Muellbauer 1980a, p. 313):

$$x_{it}(p_t, m_t) = \frac{m_t}{p_{it}} \left( \alpha_i + \sum_j \gamma_{ij} \ln p_{jt} + \beta_i \ln (m_t/P_t) \right) \quad (2)$$

where  $x_{it}$  is the consumed quantity of good  $i$ ,  $\gamma_{ij} = \frac{1}{2}(\gamma_{ij}^* + \gamma_{ji}^*)$ , and  $P_t$  is a translog price index:

$$\ln P_t = \alpha_0 + \sum_i \alpha_i \ln p_{it} + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln p_{it} \ln p_{jt} \quad (3)$$

The demand equations of the AIDS can be simplified by expressing them as expenditure shares:

$$s_{it}(p_t, m_t) = \alpha_i + \sum_j \gamma_{ij} \ln p_{jt} + \beta_i \ln (m_t/P_t) \quad (4)$$

where  $s_{it} = x_{it} p_{it}/m_t$  is the expenditure share of good  $i$ .

Microeconomic household theory and in particular the assumption of utility maximization imposes a few restrictions on the coefficients.

The "adding-up" condition ensures that the expenditure shares always sum up to one ( $\sum_i s_i = 1$ ). This condition is fulfilled if

$$\sum_i \alpha_i = 1; \sum_i \beta_i = 0; \sum_i \gamma_{ij} = 0 \forall j \quad (5)$$

The "homogeneity" condition guarantees that there is no "money illusion", i.e. if all prices and income change by the same rate (e.g. when a new currency like the EURO is introduced), the consumed quantities do not change. It is fulfilled if

$$\sum_j \gamma_{ij} = 0 \forall i \quad (6)$$

The "symmetry" condition follows from applying Shepard's Lemma to the expenditure function of the AIDS (1). It can be derived directly from the definition of the  $\gamma$ s, which can be found below equation (2):

$$\gamma_{ij} = \gamma_{ji} \forall i, j \quad (7)$$

## 2.1. Econometric estimation

To facilitate the econometric estimation of the Marshallian demand equations (4), we substitute the observed (stochastic) budget shares  $w_{it}$  for the (unobservable) deterministic budget shares  $s_{it}$  and add disturbance terms  $u_{it}$ . Hence, we get a system of equations that can be econometrically estimated:

$$w_{it} = \alpha_i + \sum_j \gamma_{ij} \ln p_{jt} + \beta_i \ln (m_t/P_t) + u_{it} \quad (8)$$

Given the fact that the observed budget shares always sum-up to one ( $\sum_i w_{it} = 1 \forall t$ ), the adding-up restrictions (5) are automatically fulfilled (Deaton and Muellbauer 1980b, p. 76) and the disturbance terms always sum-up to zero ( $\sum_i u_{it} = 0 \forall t$ ). Hence, the covariance matrix of the disturbance terms is singular (Blanciforti and Green 1983, p. 512). To avoid estimation problems due to this singularity, one of the equations has to be dropped from the system equations (8). The coefficients of the dropped equation can be calculated by the adding-up restrictions (5). Since no information is lost by dropping one equation from the system, the estimation results do not depend on what equation is dropped (Barten 1969, p. 25).

Since the Marshallian demand equations (8) are linear except for the translog price index  $P_t$ , Deaton and Muellbauer (1980a) proposed to approximate the translog price index by the Stone index. As this approximation makes the demand equations linear in parameters, the system of share equations is much easier to estimate. This simplification of the original AIDS model is commonly called “linear approximation of the AIDS” (LA-AIDS):

$$w_{it} = \alpha_i^S + \sum_j \gamma_{ij}^S \ln p_{jt} + \beta_i^S \ln \left( x_t / P_t^S \right) + u_{it}^S \quad (9)$$

where

$$\ln P_t^S = \sum_i w_{it} \ln p_{it} \quad (10)$$

and  $\alpha^S$ ,  $\beta^S$  and  $\gamma^S$  are the coefficients of the LA-AIDS.

## 2.2. Using function aidsEst

The function `aidsEst` that is included in the **micEconAids** package provides a simple interface for the econometric estimation of the AIDS. Internally, this function uses the R package **systemfit** (Henningsen and Hamann 2007a,b) to estimate the system of demand equations (4 or 9) by “Seemingly Unrelated Regression” (SUR, Zellner 1962). The `aidsEst` function is generally called by

```
> aidsEst( priceNames, shareNames, totExpName, data )
```

There are four mandatory arguments: `priceNames` must be a vector of strings that specifies the variable names of the prices, `shareNames` must be a vector of strings that specifies the variable names of the expenditure shares, `totExpName` must be a string that specifies the variable name of total expenditure, and `data` specifies a data frame that contains these variables. The most important optional arguments are `method` and `priceIndex`: `method` specifies the estimation method of the AIDS (e.g., "LA" for the LA-AIDS, equation 9) and `priceIndex` specifies the price index for the LA-AIDS (e.g., "S" for the Stone price index, equation 10).

We will demonstrate the usage of `aidsEst` by analyzing U.S. food demand using annual consumption data that are published in Blanciforti, Green, and King (1986). The data set `Blanciforti86` that is included in the R package **micEconAids** provides these data.

We start this analysis by loading the **micEconAids** package and the data set `Blanciforti86`.

```
> library(micEconAids)
> data( "Blanciforti86" )
```

This data set distinguishes four categories of food: (i) meats, (ii) fruits and vegetables, (iii) cereal and bakery products, and (iv) miscellaneous foods. The variables that are needed for this analysis are the prices of the four categories (`pFood1`, `pFood2`, `pFood3`, `pFood4`), the expenditure shares of these categories (`wFood1`, `wFood2`, `wFood3`, `wFood4`), and total expenditure on food (`xFood`). The data on food consumption are available for the first 32 years of this data set only, i.e. for the years 1947 to 1978. To avoid warning messages of removed observations due to non-available values, we remove all observations with non-available values from the data set.

```
> Blanciforti86 <- Blanciforti86[ 1:32, ]
```

For convenience, we specify two vectors that contain the variable names of the prices and the expenditure shares.

```
> priceNames <- c( "pFood1", "pFood2", "pFood3", "pFood4" )
> shareNames <- c( "wFood1", "wFood2", "wFood3", "wFood4" )
```

Now, we can use `aidsEst` to estimate the LA-AIDS model with this data set. Here, we can omit argument `method`, because it is "LA" (LA-AIDS) by default.

```
> laaidsResult <- aidsEst( priceNames, shareNames, "xFood", data = Blanciforti86,
+   priceIndex = "S" )
> print( laaidsResult )
```

```
Demand analysis with the Almost Ideal Demand System (AIDS)
Estimation Method: Linear Approximation (LA) with Stone Index (S)
Coefficients:
```

```
alpha
```

```
      wFood1      wFood2      wFood3      wFood4
-0.2472983  0.1092491  0.2682384  0.8698108
```

```
beta
```

```
      wFood1      wFood2      wFood3      wFood4
 0.32398918  0.05586316 -0.07862610 -0.30122624
```

```
gamma
```

```
      pFood1      pFood2      pFood3      pFood4
wFood1  0.10415020 -0.139880152 -0.011561838  0.047291789
wFood2 -0.13988015  0.156908650  0.003472719 -0.020501216
wFood3 -0.01156184  0.003472719  0.012489861 -0.004400742
wFood4  0.04729179 -0.020501216 -0.004400742 -0.022389831
```

This output presents all estimated coefficients: all  $\alpha$ s,  $\beta$ s, and  $\gamma$ s. Their standard errors, t-values and p-values as well as the  $R^2$  values of the expenditure shares and of the demanded quantities can be obtained by applying the `summary` method.

```
> summary( laaidsResult )
```

```
Demand analysis with the Almost Ideal Demand System (AIDS)
Estimation Method: Linear Approximation (LA) with Stone Index (S)
```

## Estimated Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
alpha 1	-0.2472983	0.0700800	-3.5288	0.0006799	***
alpha 2	0.1092491	0.0578750	1.8877	0.0625227	.
alpha 3	0.2682384	0.0344134	7.7946	1.561e-11	***
alpha 4	0.8698108	0.0905784	9.6029	3.636e-15	***
beta 1	0.3239892	0.0410414	7.8942	9.873e-12	***
beta 2	0.0558632	0.0335994	1.6626	0.1001146	
beta 3	-0.0786261	0.0200257	-3.9263	0.0001761	***
beta 4	-0.3012262	0.0529871	-5.6849	1.849e-07	***
gamma 1 1	0.1041502	0.0208044	5.0062	3.019e-06	***
gamma 1 2	-0.1398802	0.0153034	-9.1404	3.095e-14	***
gamma 1 3	-0.0115618	0.0098167	-1.1778	0.2422133	
gamma 1 4	0.0472918	0.0239750	1.9725	0.0518372	.
gamma 2 1	-0.1398802	0.0153034	-9.1404	3.095e-14	***
gamma 2 2	0.1569086	0.0286341	5.4798	4.369e-07	***
gamma 2 3	0.0034727	0.0175528	0.1978	0.8436451	
gamma 2 4	-0.0205012	0.0236281	-0.8677	0.3880508	
gamma 3 1	-0.0115618	0.0098167	-1.1778	0.2422133	
gamma 3 2	0.0034727	0.0175528	0.1978	0.8436451	
gamma 3 3	0.0124899	0.0162416	0.7690	0.4440475	
gamma 3 4	-0.0044007	0.0135413	-0.3250	0.7459983	
gamma 4 1	0.0472918	0.0239750	1.9725	0.0518372	.
gamma 4 2	-0.0205012	0.0236281	-0.8677	0.3880508	
gamma 4 3	-0.0044007	0.0135413	-0.3250	0.7459983	
gamma 4 4	-0.0223898	0.0382912	-0.5847	0.5603000	

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

R-squared Values of expenditure shares:

wFood1	wFood2	wFood3	wFood4
0.6271010	0.8315477	0.4115280	0.6569576

R-squared Values of quantities:

q_wFood1	q_wFood2	q_wFood3	q_wFood4
0.9349739	0.8463000	0.6769099	0.6322773

### 2.3. Problems and possible solutions of the LA-AIDS

The empirical application of the LA-AIDS is rather simple, because it can be estimated as a system of linear equations, which can be conveniently done in many econometrics packages. The only challenge might be the imposition of the cross-equation parameter restrictions to ensure symmetry (7). However, the linear approximation of the AIDS introduces several problems. In the following, we will discuss some of these problems as well as a few solutions that have been suggested to circumvent these problems.

#### *Simultaneity Bias*

Since the Stone index includes current budget shares ( $w_{it}$ ), they appear both on the left- and

on the right-hand side of the estimated share equation of the LA-AIDS (9). The resulting correlation between the regressor  $\ln(x_t/P_t^S)$  and the disturbance term  $u_{it}^S$  leads to a simultaneity bias of the estimation (Eales and Unnevehr 1988, p. 522). Therefore, several scholars (e.g. Blanciforti *et al.* 1986; Eales and Unnevehr 1988) use lagged shares in the Stone price index.

$$\ln P_t^{SL} = \sum_i w_{it-1} \ln p_{it} \quad (11)$$

The LA-AIDS with the Stone price index based on lagged shares will be estimated by `aidsEst`, if the price index specified in argument `method` is set to the abbreviation `SL`.

```
> laaidsResultSL <- aidsEst( priceNames, shareNames, "xFood", data = Blanciforti86,
+   priceIndex = "SL" )
```

Alternatively, Hahn (1994, p. 973) proposes to overcome this problem by using a different price index that either does not depend on current shares (e.g., the log-linear analog of the Laspeyres price index presented in the following section).

A third solution is an econometric estimation by the “Three-Stage Least Squares” (3SLS) method (Alston, Foster, and Green 1994; Buse 1993, 1994; Buse and Chan 2000). A reasonable choice of instrumental variables would be (at least) all logged prices and logged (nominal) total expenditure. The argument `instNames` of command `aidsEst` can be used to specify the names of the instrumental variables. If this argument is used, the (default) estimation method is 3SLS.

```
> instNames <- c( paste( "l", priceNames, sep = "" ), "lxFood" )
> for( i in 1:4 ){
+   Blanciforti86[[ instNames[ i ] ]] <-
+     log( Blanciforti86[[ priceNames[ i ] ]] )
+ }
> Blanciforti86[[ "lxFood" ]] <- log( Blanciforti86[[ "xFood" ]] )
> laaidsResultIV <- aidsEst( priceNames, shareNames, "xFood",
+   data = Blanciforti86, priceIndex = "S", instNames = instNames )
```

### *Units of measurement*

The Stone price index is not invariant to changes in the units of measurement, i.e. the results depend on the units of measurements of the prices (e.g. if they are in EURO/kg or US\$/pound). As this is generally undesirable, Moschini (1995) proposes to use the log-linear analog of the Paasche (12) or Laspeyres (13) price index or the Tornqvist price index (14) in place of the Stone price index in the LA-AIDS:

$$\ln P_t^P = \sum_k w_{kt} \ln \frac{p_{kt}}{p_{k0}} \quad (12)$$

$$\ln P_t^L = \sum_k w_{k0} \ln \frac{p_{kt}}{p_{k0}} \quad (13)$$

$$\ln P_t^T = \frac{1}{2} \sum_k \left( (w_{kt} + w_{k0}) \ln \frac{p_{kt}}{p_{k0}} \right) \quad (14)$$

where  $w_{k0}$  and  $p_{k0}$  indicate expenditure shares and prices in a “base” period (e.g. at sample mean values).

The function `aidsEst` estimates the LA-AIDS with the Paasche, Laspeyres or Tornqvist price index, if argument `priceIndex` is set to the abbreviation "P", "L", or "T", respectively.

```
> laaidsResultP <- aidsEst( priceNames, shareNames, "xFood",
+   data = Blanciforti86, priceIndex = "P" )
> laaidsResultL <- aidsEst( priceNames, shareNames, "xFood",
+   data = Blanciforti86, priceIndex = "L" )
> laaidsResultT <- aidsEst( priceNames, shareNames, "xFood",
+   data = Blanciforti86, priceIndex = "T" )
```

Since the weights  $w_{k0}$  of the Laspeyres price index (13) are constants and only invariance up to a multiplicative constant is required, this price index can be simplified (Moschini 1995, p. 65).

$$\ln P_t^{Ls} = \sum_k w_{k0} \ln p_{kt} \quad (15)$$

The LA-AIDS with this simplified Laspeyres price index is estimated by `aidsEst`, if argument `priceIndex` is equal to "Ls". This is its default value of this argument, and hence, it can be omitted.

```
> laaidsResultLs <- aidsEst( priceNames, shareNames, "xFood",
+   data = Blanciforti86 )
```

Since (13) differs from (15) only by an additive constant, this model is equivalent to the model with the (non-simplified) Laspeyres price index. This simplification changes only the  $\alpha$ s but has no effect on the  $\beta$ s,  $\gamma$ s, and  $R^2$  values

```
> all.equal( coef( laaidsResultL )$alpha, coef( laaidsResultLs )$alpha )
```

```
[1] "Mean relative difference: 0.707"
```

```
> all.equal( coef( laaidsResultL )$beta, coef( laaidsResultLs )$beta )
```

```
[1] TRUE
```

```
> all.equal( coef( laaidsResultL )$gamma, coef( laaidsResultLs )$gamma )
```

```
[1] TRUE
```

```
> all.equal( laaidsResultL$r2, laaidsResultLs$r2 )
```

```
[1] TRUE
```

While all these alternative price indices — unlike the Stone index — perform very well in the simulation study of Moschini (1995), the Monte Carlo studies of Buse (1998) and Buse and Chan (2000) indicate that the Paasche index is as poor as the Stone index. An advantage



of the Laspeyres index is that the actual budget shares do not appear on the right-hand side of the share equation, which avoids the simultaneity problem (Asche and Wessells 1997, p. 1183).

### *Approximation errors*

The approximation of the true translog price index necessarily leads to an approximation error. Deaton and Muellbauer (1980a, p. 316) argue that if prices are closely collinear, any price index (e.g.  $P^S$ ) is an adequate approximation to the true translog price index  $P$ . However, Buse (1998, p. 210) shows that high collinearity is neither necessary nor sufficient for a good approximation. The approximation error can be seen as an “omitted variables problem” (Pashardes 1993, p. 909) or an “errors-in-variables problem” (Buse 1994, p. 782). Both perspectives generally lead to biased and inconsistent OLS and SUR estimates. Generally, an instrumental variable (IV) estimation might solve this problem, but Buse (1994, p. 782f) and Buse and Chan (2000, p. 520) show that it is impossible to find appropriate instrumental variables (IV) in this case. Therefore, this problem remains unsolved, and hence, no solution can be presented in **micEconAids**.

### *Theoretical inconsistency*

In the LA-AIDS — like in the original AIDS — adding-up (5) is automatically fulfilled and homogeneity can be imposed by parameter restrictions (6), while the usual symmetry restrictions (7) guarantee symmetry only if all prices are identical (Hahn 1994, p. 975). Therefore, the LA-AIDS itself is not an integrable demand system (Alston *et al.* 1994, p. 351). Of course, no econometric software can solve this problem. Therefore, the question arises whether the linear approximation should be applied at all.

## **2.4. Iterative Linear Least Squares Estimation**

As no solution exists for some of the severe problems of the LA-AIDS, the estimation of the non-linear AIDS suggests itself. It can be consistently and asymptotically efficiently estimated by full information maximum likelihood (FIML) or by non-linear seemingly unrelated regression (SUR). Although non-linear estimation methods are implemented in most econometric software packages, these procedures often cause problems in practical work (e.g. non-convergence). Moreover, while the R package **systemfit** (Henningsen and Hamann 2007a,b) has proved to be reliable in linear system estimations for a long time, its function for non-linear system estimations (**nlsystemfit**) is still under development. However, the non-linear AIDS can also be estimated by iterations of linear estimations (Browning and Meghir 1991; Michalek and Keyzer 1992; Blundell and Robin 1999). In a first step, the share equations can be estimated by linear estimation techniques holding the translog price index fixed. In a second step, the translog price index can be updated with the newly estimated coefficients. These two steps can be repeated until the coefficients converge. Initial values for the coefficients to calculate the translog price index can be obtained from a LA-AIDS estimation. Blundell and Robin (1999) prove the convergence of this iterated estimator and name it “Iterated Linear Least Squares Estimator” (ILLE). As the translog price index is not a fixed regressor but depends on stochastic estimated coefficients, the standard coefficient variance covariance matrix obtained from the regression in the last iteration is not appropriate. Blundell and

Robin (1999) derive the asymptotic variance covariance matrix of the estimated coefficients:

$$\widehat{\text{VAR}}(\hat{\theta}) = \hat{J}^{-1} \left( \hat{\Sigma} \otimes \left( G(\hat{\theta})' G(\hat{\theta}) \right) \right) \left( \hat{J}' \right)^{-1} \quad (16)$$

where

$$\hat{J} = \left( I_N \otimes G(\hat{\theta})' \right) \nabla(\hat{\theta}) \quad (17)$$

$\theta$  is a vector of all  $\alpha$ ,  $\beta$  and  $\gamma$  coefficients,  $\hat{\Sigma}$  is the residual covariance matrix,  $I$  is an identity matrix,  $G(\hat{\theta})$  is the matrix of the constructed regressors for a (conditional) linear estimation given the estimated coefficients, and  $\nabla(\hat{\theta}) = \partial([I_N \otimes G(\hat{\theta})]\hat{\theta})/\partial\theta'$  is the Jacobian matrix of the expenditure shares with respect to coefficients evaluated at their estimated values.

If argument `method` of `aidsEst` is "IL", this function estimates the non-linear AIDS by the "Iterated Linear Least Squares Estimator" (ILLE).

```
> aidsResult <- aidsEst( priceNames, shareNames, "xFood",
+   data = Blanciforti86, method = "IL" )
```

In this case, argument `priceIndex` can be used to specify the price index that is used for the LA-AIDS estimation to obtain initial values (see section 2.2). However, the results of the ILLE estimation generally do not depend on the choice of the price index to obtain the initial values.

### 3. Consistency with economic theory

Microeconomic household theory requires several properties of the expenditure function (1) and the corresponding demand functions (2). For the AIDS, the adding-up conditions (5) are automatically fulfilled, while the homogeneity (6) and symmetry (7) restrictions can be imposed globally by parameter restrictions. However, the monotonicity and concavity properties generally cannot be imposed by parameter restrictions, but can be checked after the estimation.

#### 3.1. Adding-up, homogeneity and symmetry

The following commands verify that the adding-up conditions (5) are (automatically) fulfilled.

```
> all.equal( sum( coef( aidsResult )$alpha ), 1 )
```

```
[1] TRUE
```

```
> all.equal( sum( coef( aidsResult )$beta ), 0 )
```

```
[1] TRUE
```

```
> all.equal( colSums( coef( aidsResult )$gamma ), rep( 0, 4 ),
+   check.attributes = FALSE )
```

```
[1] TRUE
```

By default, `aidsEst` imposes parameter restrictions to ensure homogeneity (6) and symmetry (7). The following checks verify that these conditions are fulfilled.

```
> all.equal( rowSums( coef( aidsResult )$gamma ), rep( 0, 4 ),
+   check.attributes = FALSE )
```

```
[1] TRUE
```

```
> isSymmetric( coef( aidsResult )$gamma, tol = 1e-10, check.attributes = FALSE )
```

```
[1] TRUE
```

If the user does not want to impose homogeneity and/or symmetry, she can set the optional arguments `hom` and/or `sym` to `FALSE`. Since symmetry (7) together with adding-up (5) implies homogeneity (6), it is not possible to impose symmetry without homogeneity. However, the AIDS can be estimated with homogeneity but without symmetry imposed.

```
> aidsResultHom <- aidsEst( priceNames, shareNames, "xFood", data = Blanciforti86,
+   method = "IL", sym = FALSE )
> isSymmetric( coef( aidsResultHom )$gamma, tol = 1e-10, check.attributes = FALSE )
```

```
[1] FALSE
```

In this case, the symmetry conditions (7) are no longer fulfilled. These restrictions can be statistically tested, for instance by a Likelihood-Ratio test that is implemented in the `lrtest` method for objects of class `aidsEst`.

```
> lrtest( aidsResult, aidsResultHom )
```

Likelihood ratio test

Model 1: `aidsResult` (AIDS, symmetry and homogeneity imposed)

Model 2: `aidsResultHom` (AIDS, homogeneity imposed)

	#Df	LogLik	Df	Chisq	Pr(>Chisq)
1	18	359.63			
2	21	362.35	3	5.4452	0.142

The LR test does not reject the Null hypothesis. Hence, we can conclude that our model and data set do not contradict the symmetry condition.

Now, we can additionally relax the homogeneity restriction.

```
> aidsResultUnr <- aidsEst( priceNames, shareNames, "xFood", data = Blanciforti86,
+   method = "IL", hom = FALSE, sym = FALSE )
> all.equal( rowSums( coef( aidsResultUnr )$gamma ), rep( 0, 4 ),
+   check.attributes = FALSE )
```

```
[1] "Mean relative difference: 1"
```

In this case, also the homogeneity conditions (6) are no longer fulfilled. Again, we can statistically test these restrictions.

```
> lrtest( aidsResultHom, aidsResultUnr )
```

Likelihood ratio test

Model 1: aidsResultHom (AIDS, homogeneity imposed)

Model 2: aidsResultUnr (AIDS, unrestricted)

	#Df	LogLik	Df	Chisq	Pr(>Chisq)
1	21	362.35			
2	24	375.46	3	26.232	8.526e-06 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

This Null hypothesis is clearly rejected. Thus, we can conclude that our model and data set contradict the homogeneity condition.

It is also possible to test the symmetry and homogeneity restrictions simultaneously.

```
> lrtest( aidsResult, aidsResultUnr )
```

Likelihood ratio test

Model 1: aidsResult (AIDS, symmetry and homogeneity imposed)

Model 2: aidsResultUnr (AIDS, unrestricted)

	#Df	LogLik	Df	Chisq	Pr(>Chisq)
1	18	359.63			
2	24	375.46	6	31.678	1.881e-05 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Also this Null hypothesis is clearly rejected meaning that homogeneity and symmetry are not simultaneously satisfied.

### 3.2. Monotonicity

The “monotonicity” condition says that the expenditure function must be monotonically increasing in prices, which implies that its first derivatives with respect to prices are always non-negative. According to Shepard’s Lemma, this can be checked by testing whether the quantities and expenditure shares in equations (2) and (4) are non-negative. The check for monotonicity of the AIDS is implemented in function `aidsMono`. It is generally called by

```
> aidsMono( priceNames, totExpName, coef, data )
```

There are four mandatory arguments: `priceNames` must be a vector of strings that specifies the variable names of the prices, `totExpName` must be a string that specifies the variable name of total expenditure, `coef` must be a list that contains the coefficients, i.e. a scalar `alpha0`,

a vector `alpha`, a vector `beta`, and a matrix `gamma`, and `data` must be a data frame that contains the prices and total expenditure.

We demonstrate the usage of `aidsMono` by checking whether the monotonicity condition is fulfilled in the AIDS model estimated in Section 2.4.

```
> aidsMono( priceNames, "xFood", coef = coef( aidsResult ),
+   data = Blanciforti86 )
```

```
Checking the monotonicity condition of an Almost Ideal Demand System (AIDS):
Monotonicity is fulfilled at 32 out of 32 observations (100%)
```

The output shows that monotonicity is satisfied for all goods at all observations.

Function `aidsMono` can check also LA-AIDS models. Although it does the same checks as for the AIDS (non-negativity of the expenditure shares), the results have to be interpreted differently, because no expenditure function for the LA-AIDS exists. Argument `priceIndex` can be used to specify a price index other than the translog index. If the Paasche, (simplified) Laspeyres, or Tornqvist index is specified, arguments `basePrices` and/or `baseShares` have to be used to specify the base prices and base expenditure shares, respectively.

```
> aidsMono( priceNames, "xFood", coef = coef( laaidsResult ),
+   data = Blanciforti86, priceIndex = "S" )
```

```
Checking the monotonicity condition of an Linear Approximate Almost Ideal Demand System (L
Monotonicity is fulfilled at 32 out of 32 observations (100%)
```

```
> aidsMono( priceNames, "xFood", coef = coef( laaidsResultT ),
+   data = Blanciforti86, priceIndex = "T",
+   basePrices = laaidsResultT$basePrices,
+   baseShares = laaidsResultT$baseShares )
```

```
Checking the monotonicity condition of an Linear Approximate Almost Ideal Demand System (L
Monotonicity is fulfilled at 32 out of 32 observations (100%)
```

If argument `priceIndex` specifies a price index of the LA-AIDS, “fitted” values are used for current and lagged expenditure shares in these price indices (see Section 5). However, if the user wants to use any other weights for the calculation of the price indices (e.g. observed expenditure shares), she can set argument `priceIndex` to a numeric vector containing the log values of the desired price index (e.g. the price index used in the estimation).

```
> aidsMono( priceNames, "xFood", coef = coef( laaidsResult ),
+   data = Blanciforti86, priceIndex = laaidsResult$lnp )
```

```
Checking the monotonicity condition of an Linear Approximate Almost Ideal Demand System (L
Monotonicity is fulfilled at 32 out of 32 observations (100%)
```

```
> aidsMono( priceNames, "xFood", coef = coef( laaidsResultT ),
+   data = Blanciforti86, priceIndex = laaidsResultT$lnp )
```

Checking the monotonicity condition of an Linear Approximate Almost Ideal Demand System (LAI) :  
 Monotonicity is fulfilled at 32 out of 32 observations (100%)

### 3.3. Concavity

The “concavity” condition guarantees that there is a unique solution to the utility maximization problem. The expenditure function (1) is concave, if its Hessian matrix — which in this context is also called “Slutsky substitution matrix” — is negative semidefinite. The  $i,j$ th element of the substitution matrix is:

$$\frac{\partial e(p_t, U_t)^2}{\partial p_{it} \partial p_{jt}} = \frac{m_t}{p_{it} p_{jt}} c_{ijt} \quad (18)$$

where

$$c_{ijt} = \gamma_{ij} + \beta_i \beta_j \ln(m_t / P_t) + s_{it} s_{jt} - \delta_{ij} s_{it} \quad (19)$$

and  $\delta_{ij}$  is Kronecker delta with  $\delta_{ij} = 1 \forall i = j$  and  $\delta_{ij} = 0 \forall i \neq j$ . To check the concavity of the expenditure function (1), it is sufficient to test for negative semidefiniteness of the matrices  $C_t = [c_{ijt}]$ , because it is a necessary and sufficient condition for the negative semidefiniteness of the Slutsky substitution matrix (Deaton and Muellbauer 1980a, p. 316). The check for concavity of the AIDS is implemented in function `aidsConcav`. It is generally called by

```
> aidsConcav( priceNames, totExpName, coef, data )
```

The four mandatory arguments `priceNames`, `totExpName`, `coef`, and `data` are identical to the mandatory arguments of function `aidsMono`. We demonstrate the usage of `aidsConcav` by checking whether the concavity condition is fulfilled in the AIDS model estimated in Section 2.4.

```
> aidsConcav( priceNames, "xFood", coef = coef( aidsResult ),
+   data = Blanciforti86 )
```

Checking the concavity condition of an Almost Ideal Demand System (AIDS):  
 Concavity is fulfilled at 0 out of 32 observations (0%)

The output shows that concavity is satisfied not at a single observation. By default, `aidsConcav` uses fitted expenditure shares for  $s_{it}$  and  $s_{jt}$  in (19). However, if the user wants to use observed expenditure shares ( $w_{it}$ ,  $w_{jt}$ ) instead, she can use the optional argument `shareNames` to specify the variable names of the observed expenditure shares.

```
> aidsConcav( priceNames, "xFood", coef = coef( aidsResult ),
+   data = Blanciforti86, shareNames = shareNames )
```

Checking the concavity condition of an Almost Ideal Demand System (AIDS):  
 Concavity is fulfilled at 0 out of 32 observations (0%)

However, this does not change the result: concavity is satisfied not at a single observation.

If the symmetry condition of an AIDS model is not fulfilled, it is not possible to check whether its expenditure function is concave, because no corresponding expenditure function exists. Similarly, no expenditure function of the LA-AIDS exists. However, if the  $\gamma$  matrix of an LA-AIDS is symmetric, `aidsConcav` can check of course whether the expenditure function of an AIDS with coefficients of this LA-AIDS would be concave. Since the LA-AIDS does not have the coefficient  $\alpha_0$ , the user has to specify a value for this coefficient.

```
> laaidsCoef <- coef( laaidsResult )
> laaidsCoef$alpha0 <- 1
> aidsConcav( priceNames, "xFood", coef = laaidsCoef,
+   data = Blanciforti86 )
```

```
Checking the concavity condition of an Almost Ideal Demand System (AIDS):
Concavity is fulfilled at 0 out of 32 observations (0%)
```

However, it is questionable if this makes much sense.

### 3.4. All conditions

Since it can be cumbersome to check for all conditions separately, a wrapper function `aidsConsist` that checks all conditions simultaneously is included in the **micEconAids** package. It is generally called by

```
> aidsConsist( priceNames, totExpName, coef, data )
```

where the four mandatory arguments are identical to the mandatory arguments of `aidsMono` and `aidsConcav`. Furthermore, it has the optional arguments `priceIndex`, `basePrices`, and `baseShares` that are passed to `aidsMono` as well as the optional argument `shareNames` that is passed to `aidsConcav`. We demonstrate the usage of `aidsConsist` with the AIDS model estimated in Section 2.4.

```
> aidsConsist( priceNames, "xFood", coef = coef( aidsResult ),
+   data = Blanciforti86 )
```

```
Checking theoretical consistency of an Almost Ideal Demand System (AIDS):
The adding-up condition is fulfilled
The homogeneity condition is fulfilled
The symmetry condition is fulfilled
Monotonicity is fulfilled at 32 out of 32 observations (100%)
Concavity is fulfilled at 0 out of 32 observations (0%)
```

As noted above, checking concavity of the expenditure function requires that the expenditure function of the model exists. Therefore, `aidsConsist` checks the concavity condition, only if the symmetry condition is fulfilled and the translog price index is used.

Checking of theoretical consistency of an estimated AIDS or LA-AIDS model can be further simplified by the `checkConsist` method for objects of class `aidsEst`. This method extracts the required elements from an object returned by `aidsEst` and passes them to `aidsTestConsist`. The following command does the same as the previous command.

```
> aidsResultConsist <- checkConsist( aidsResult )
```

The `checkConsist` method for objects of class `aidsEst` has an optional argument `observedShares`. It determines whether fitted or observed expenditure shares are used in the price index of the LA-AIDS to check monotonicity and in the calculation of the substitution matrix to check concavity. It is `FALSE` by default, which means that fitted expenditure shares are used. If this argument is `TRUE`, observed expenditure shares are used.

```
> aidsResultConsistObs <- checkConsist( aidsResult, observedShares = TRUE )
```

## 4. Demand elasticities

As the parameters of most demand models like the AIDS do not have a straight-forward interpretation and slopes of demand curves depend on arbitrary units of measurements, economists present their results often in terms of elasticities.

### 4.1. Elasticities of the AIDS

The expenditure (income) elasticities ( $\eta_i$ ) and Marshallian (uncompensated) price elasticities ( $\Theta_{ij}$ ) of the AIDS can be derived from the Marshallian (uncompensated) demand functions (4):<sup>2</sup>

$$\eta_i = \frac{\partial x_i}{\partial m} \frac{m}{x_i} = 1 + \frac{\beta_i}{s_i} \quad (20)$$

$$\Theta_{ij} = \frac{\partial x_i}{\partial p_j} \frac{p_j}{x_i} = -\delta_{ij} + \frac{\gamma_{ij}}{s_i} - \frac{\beta_i}{s_i} \left( \alpha_j + \sum_k \gamma_{kj} \ln p_k \right) \quad (21)$$

(Anderson and Blundell 1983, p. 400). Hicksian (compensated) price elasticities ( $\Theta_{ij}^*$ ) of the AIDS can be derived from the Hicksian (compensated) demand functions or by inserting the expenditure elasticity and Marshallian price elasticity in the Slutsky equation (Deaton and Muellbauer 1980b, p. 45):

$$\Theta_{ij}^* = \Theta_{ij} + \eta_i * s_j \quad (22)$$

Function `aidsElas` can be used to calculate demand elasticities of the AIDS by (20), (21), and (22). It has one mandatory argument `coef`; the most important optional arguments are `prices`, `shares`, and `totExp`.

```
> aidsElas( coef, prices = NULL, shares = NULL, totExp = NULL )
```

Argument `coef` is a list that contains the coefficients (such as argument `coef` of `aidsMono`); `prices` and `shares` must be numerical vectors that contain the prices and expenditure shares, respectively, and `totExp` must be the total expenditure, at which the elasticities should be calculated. If the optional argument `shares` is provided, these values are used as expenditure shares  $s_i$  in 20 and 21; otherwise, fitted expenditure shares are calculated with (3) and (4) using the provided coefficients, prices, and total expenditure. In the first case, argument

<sup>2</sup>In this section we omit the indicator of time ( $t$ ) for convenience.



`totExp` is not required, because total expenditure is not needed for these calculations. The usage of `aidsElas` is demonstrated by calculating the demand elasticities of the AIDS at sample mean values.

```
> pMeans <- colMeans( Blanciforti86[ , priceNames ] )
> wMeans <- colMeans( Blanciforti86[ , shareNames ] )
> aidsResultElas <- aidsElas( coef( aidsResult ), prices = pMeans,
+   shares = wMeans )
> print( aidsResultElas )
```

Demand Elasticities (original AIDS formulas)

Expenditure Elasticities

```
q_wFood1 q_wFood2 q_wFood3 q_wFood4
  2.065    1.234    0.395    0.166
```

Marshallian (uncompensated) Price Elasticities

```
      pFood1 pFood2 pFood3 pFood4
q_wFood1 -1.012 -0.678 -0.17676 -0.198
q_wFood2 -0.790 -0.250 -0.00839 -0.186
q_wFood3  0.109  0.157 -0.82620  0.165
q_wFood4  0.415  0.110  0.09354 -0.784
```

Hicksian (compensated) Price Elasticities

```
      pFood1 pFood2 pFood3 pFood4
q_wFood1 -0.371 -0.26473  0.100  0.535
q_wFood2 -0.407 -0.00221  0.157  0.252
q_wFood3  0.232  0.23583 -0.773  0.306
q_wFood4  0.466  0.14356  0.116 -0.725
```

First, the expenditure elasticities, then the Marshallian (uncompensated) prices elasticities, and finally the Hicksian (compensated) price elasticities are printed.

Calculating these elasticities not at observed but at fitted expenditure shares has only a minor effect on the returned values.

```
> xtMean <- mean( Blanciforti86[ , "xFood" ] )
> aidsResultElas2 <- aidsElas( coef( aidsResult ), prices = pMeans,
+   totExp = xtMean )
> all.equal( aidsResultElas, aidsResultElas2 )
```

```
[1] "Component "exp": Mean relative difference: 0.0055"
[2] "Component "marshall": Mean relative difference: 0.00621"
[3] "Component "hicks": Mean relative difference: 0.00927"
```

A special feature of `aidsElas` is the approximate calculation of the covariance matrix of the elasticities. This calculation is based on the formula of Klein (1953, p. 258), which is also known as “delta method.”

$$\widehat{\text{VAR}}(\Psi(\hat{\theta})) = \nabla^*(\hat{\theta}) \widehat{\text{VAR}}(\hat{\theta}) \nabla^{*\prime}(\hat{\theta})' \quad (23)$$

where  $\Psi(\hat{\theta})$  is a vector of all demand elasticities ( $\eta, \Theta, \Theta^*$ ) and  $\nabla^*(\hat{\theta}) = \partial\Psi(\hat{\theta})/\partial\theta'$  is the Jacobian matrix of the demand elasticities with respect to coefficients evaluated at their estimated values. The covariance matrix of the coefficients, which is required for calculating the covariance matrix of the elasticities, can be specified by the optional argument `coefCov`. Furthermore, the optional argument `df` can be used to specify the degrees of freedom for calculating the marginal levels of significance (“P values”) of (approximate)  $t$  tests of the elasticities. Summary statistics of the elasticities can be obtained by applying the `summary` method to an object returned by `aidsElas`.

```
> aidsResultElasCov <- aidsElas( coef( aidsResult ), prices = pMeans,
+   totExp = xtMean, coefCov = vcov( aidsResult ),
+   df = df.residual( aidsResult ) )
> summary( aidsResultElasCov )
```

	Estimate	Std. Error	t value	Pr(> t )	
Ex q_wFood1	2.05791	0.13178	15.62	< 2e-16	***
Ex q_wFood2	1.23188	0.23737	5.19	1.4e-06	***
Ex q_wFood3	0.39237	0.23320	1.68	0.0962	.
Ex q_wFood4	0.15691	0.21251	0.74	0.4623	
Eh q_wFood1 pFood1	-0.36856	0.06484	-5.68	1.9e-07	***
Eh q_wFood1 pFood2	-0.25723	0.05189	-4.96	3.7e-06	***
Eh q_wFood1 pFood3	0.09912	0.03381	2.93	0.0043	**
Eh q_wFood1 pFood4	0.52666	0.07642	6.89	9.5e-10	***
Eh q_wFood2 pFood1	-0.39698	0.08013	-4.95	3.7e-06	***
Eh q_wFood2 pFood2	-0.00797	0.17874	-0.04	0.9645	
Eh q_wFood2 pFood3	0.15613	0.11453	1.36	0.1764	
Eh q_wFood2 pFood4	0.24882	0.13725	1.81	0.0734	.
Eh q_wFood3 pFood1	0.23204	0.07886	2.94	0.0042	**
Eh q_wFood3 pFood2	0.23683	0.17368	1.36	0.1763	
Eh q_wFood3 pFood3	-0.77299	0.13394	-5.77	1.3e-07	***
Eh q_wFood3 pFood4	0.30411	0.11406	2.67	0.0092	**
Eh q_wFood4 pFood1	0.46809	0.06786	6.90	9.2e-10	***
Eh q_wFood4 pFood2	0.14330	0.07922	1.81	0.0741	.
Eh q_wFood4 pFood3	0.11546	0.04356	2.65	0.0096	**
Eh q_wFood4 pFood4	-0.72685	0.12420	-5.85	9.1e-08	***
Em q_wFood1 pFood1	-1.01159	0.06411	-15.78	< 2e-16	***
Em q_wFood1 pFood2	-0.67390	0.06280	-10.73	< 2e-16	***
Em q_wFood1 pFood3	-0.17557	0.04036	-4.35	3.8e-05	***
Em q_wFood1 pFood4	-0.19684	0.09274	-2.12	0.0367	*
Em q_wFood2 pFood1	-0.78191	0.07940	-9.85	1.2e-15	***
Em q_wFood2 pFood2	-0.25740	0.21043	-1.22	0.2247	
Em q_wFood2 pFood3	-0.00830	0.11766	-0.07	0.9439	
Em q_wFood2 pFood4	-0.18428	0.12428	-1.48	0.1419	
Em q_wFood3 pFood1	0.10944	0.10585	1.03	0.3042	
Em q_wFood3 pFood2	0.15739	0.20409	0.77	0.4428	
Em q_wFood3 pFood3	-0.82536	0.12891	-6.40	8.3e-09	***
Em q_wFood3 pFood4	0.16616	0.10024	1.66	0.1011	

```

Em q_wFood4 pFood1  0.41906    0.08728    4.80  6.8e-06 ***
Em q_wFood4 pFood2  0.11153    0.10518    1.06  0.2921
Em q_wFood4 pFood3  0.09452    0.06114    1.55  0.1259
Em q_wFood4 pFood4 -0.78201    0.12145   -6.44  7.1e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

The row names indicate the elasticities, where “Ex” indicates expenditure (income) elasticities, “Eh” indicates Hicksian (compensated) price elasticities and “Em” indicates Marshallian (uncompensated) price elasticities.

Calculating the demand elasticities of an estimated AIDS model can be simplified by using the `elas` method for objects of class `aidsEst`. This is a convenient tool that extracts the required elements from the provided object and passes them to `aidsElas`.

```

> aidsResultElasCov2 <- elas( aidsResult )
> all.equal( aidsResultElasCov, aidsResultElasCov2 )

```

```
[1] TRUE
```

The `elas` method for objects of class `aidsEst` has an optional argument `observedShares`. It determines whether fitted or observed expenditure shares are used in the calculations of the price indices. It is `FALSE` by default, which means that fitted expenditure shares are used. If this argument is `TRUE`, observed expenditure shares are used.

```

> aidsResultElasCovObs <- elas( aidsResult, observedShares = TRUE )

```

## 4.2. Elasticities of the LA-AIDS with Stone price index

Several authors have shown that the estimated parameters of the LA-AIDS are not necessarily close to the AIDS parameters. Therefore, the formulas to calculate the demand elasticities of the (non-linear) AIDS (20, 21, 22) are generally not appropriate to calculate demand elasticities based on parameters from the LA-AIDS. To overcome this problem, some scholars have derived special formulas to calculate demand elasticities of the LA-AIDS with the Stone price index. Generally, the Marshallian price elasticities and expenditure elasticities of the LA-AIDS (and also of the AIDS) can be obtained by (Green and Alston 1990, p. 443):

$$\Theta_{ij} = -\delta_{ij} + \frac{\gamma_{ij}^S}{w_i} - \frac{\beta_i^S}{w_i} \frac{\partial \ln P^S}{\partial \ln p_j} \quad (24)$$

$$\eta_i = 1 + \frac{\beta_i}{w_i} \left( 1 - \frac{\partial \ln P^S}{\partial \ln x} \right) \quad (25)$$

The differences between the various formulas for elasticities that several authors have suggested for the LA-AIDS mainly come from the different treatment of  $\partial \ln P^S / \partial \ln p_j$  (Green and Alston 1990, p. 443) and  $\partial \ln P^S / \partial \ln x$ .

Green and Alston (1990) elaborated  $\partial \ln P^S / \partial \ln p_j$  and  $\partial \ln P^S / \partial \ln x$  in detail and proposed following formulas:

$$\Theta_{ij}^{GA} = -\delta_{ij} + \frac{\gamma_{ij}^S}{w_i} - \frac{\beta_i^S}{w_i} \left( w_j + \sum_k w_k \ln p_k \left( \Theta_{kj}^{GA} + \delta_{kj} \right) \right) \quad (26)$$

$$\eta_i^{GA} = 1 + \frac{\beta_i^S}{w_i} \left( 1 - \sum_j w_j \ln p_j \left( \eta_j^{GA} - 1 \right) \right) \quad (27)$$

The drawback of these formulas is that  $\Theta_{ij}^{GA}$  and  $\eta_i^{GA}$  appear also on the right-hand side, which complicates their calculation. However, Buse (1994, p. 784) derives formulas that are equivalent to (26) and (27) but do not have the drawback of having  $\Theta_{ij}$  or  $\eta_i$  on the right-hand side:<sup>3</sup>

$$\Theta_{ij}^{B1} = -\delta_{ij} + \frac{\gamma_{ij}^S}{w_i} - \frac{\beta_i^S}{w_i} \left( \frac{w_j + \sum_k \gamma_{kj}^S \ln p_k}{1 + \sum_k \beta_k^S \ln p_k} \right) \quad (28)$$

$$\eta_i^{B1} = 1 + \frac{\beta_i^S}{w_i (1 + \sum_k \beta_k^S \ln p_k)} \quad (29)$$

The function `aidsElas` can calculate demand elasticities not only for the AIDS, but also for the LA-AIDS. By default `aidsElas` uses formulas (3) and (4) to calculate fitted expenditure shares and formulas (20), (21), and (22) to calculate demand elasticities. However, the optional arguments `priceIndex` and `method` can be used to specify different price indices and different formulas for demand elasticities that are more suitable for the LA-AIDS. For instance, if argument `priceIndex` is "S" and argument `method` is "B1" or "GA", fitted shares are calculated with the Stone price index and the first set of Buse's 1994 formulas (28, 29), which are equivalent to the formulas of Green and Alston (1990) (26, 27), are applied to calculate demand elasticities.

```
> laaidsResultElasB1 <- aidsElas( coef( laaidsResult ),
+   prices = pMeans, totExp = xtMean, method = "B1",
+   priceIndex = "S" )
```

Again, this command can be simplified by using the `elas` method for objects of class `aidsEst`.

```
> laaidsResultElasB1a <- elas( laaidsResult, method = "B1" )
> all.equal( laaidsResultElasB1, laaidsResultElasB1a )
```

[1] TRUE

Alternatively, Buse (1994, p. 784) treats  $\partial \ln P^S / \partial p_j$  as an approximation of  $\partial \ln P / \partial p_j$  and replaced  $w_k$  in  $\partial \ln w_k / \partial \ln p_j$  by the right hand-side of the share equation (8) to get the

---

<sup>3</sup>Buse (1994, p. 785) calculates the expenditure elasticities by  $\eta_i^{B1} = 1 + \frac{\beta_i^S}{w_i} \left( 1 - \frac{\sum_k \beta_k^* \ln p_k}{1 + \sum_k \beta_k^S \ln p_k} \right)$ . This formula is identical to equation (29).

following formulas:

$$\Theta_{ij}^{B2} = -\delta_{ij} + \frac{\gamma_{ij}^S}{w_i} - \frac{\beta_i^S}{w_i} \left( w_j + \sum_k \gamma_{kj}^S \ln p_k - \sum_k \beta_k^S \ln p_k \left( \alpha_j + \sum_l \gamma_{lj}^S \ln p_l \right) \right) \quad (30)$$

$$\eta_i^{B2} = 1 + \frac{\beta_i^S}{w_i} \left( 1 - \sum_k \beta_k^S \ln p_k \right) \quad (31)$$

If argument `method` of `aidsElas` is set to "B2", Buse's 1994 second set of formulas (28, 29) is used to calculate the elasticities.

```
> laaidsResultElasB2 <- elas( laaidsResult, method = "B2" )
```

Goddard (1983) and Chalfant (1987)<sup>4</sup> assume the budget shares to be independent of prices and derive the following formula:

$$\Theta_{ij}^{Go} = -\delta_{ij} + \frac{\gamma_{ij}^S}{w_i} - \frac{\beta_i^S w_j}{w_i} \quad (32)$$

Assuming that the budget shares are independent of total expenditure, one gets the original formula for expenditure elasticities in the AIDS (20). If `aidsElas` is called with argument `method` equal to "Go" or "Ch", the elasticities are calculated with the formulas of Goddard (1983) and Chalfant (1987) (32, 20).

```
> laaidsResultElasGo <- elas( laaidsResult, method = "Go" )
```

Eales and Unnevehr (1988) calculate demand elasticities of the LA-AIDS by

$$\Theta_{ij}^{EU} = -\delta_{ij} + \frac{\gamma_{ij}^S}{w_i} \quad (33)$$

This formula is correct only in the special cases if preferences are homothetic ( $\beta_i = 0 \forall i$ ) or if the Stone price index  $P^S$  is independent of individual prices (Green and Alston 1990, p. 443). If argument `method` of `aidsElas` is "EU", the formula of Eales and Unnevehr (33) is used to calculate the Marshallian price elasticities, while the expenditure elasticities are calculated by equation (20).

```
> laaidsResultElasEU <- elas( laaidsResult, method = "EU" )
```

When all prices are unity (Buse 1994, p. 784) or in the case of (imposed) homogeneity when all prices are equal,  $\Theta_{ij}^{GA,B1}$ ,  $\Theta_{ij}^{B2}$  and  $\Theta_{ij}^{GC}$  are identical. The same holds for  $\eta_i$ ,  $\eta_i^{GA,B1}$ , and  $\eta_i^{B2}$ .

Currently, `aidsEla` can calculate the covariance matrix only for elasticities that are calculated by the formulas for the non-linear AIDS model, i.e. (20), (21), and (22).

### 4.3. Elasticities of the LA-AIDS with alternative price indices

<sup>4</sup>There is some disagreement in the literature who was the first who applied this formula, since Goddard (1983) applied this formula to a LA-AIDS with Paasche index and Chalfant (1987) was the first who applied this formula to the LA-AIDS with Stone index.

The formulas for calculating demand elasticities of the LA-AIDS model with the Stone price index are not necessarily suitable for LA-AIDS models with alternative price indices. One exception is the approach of [Eales and Unnevehr \(1988\)](#), i.e. (33) and (20). Their formulas ignore any connection between the price index and prices or total expenditure. Therefore, this approach yields the same formulas for all price indices. If argument `method` is "EU", `aidsElas` uses formulas (33) and (20) no matter which price index is specified by argument `priceIndex`. Further approaches for calculating demand elasticities of LA-AIDS models with alternative price indices are presented in the following paragraphs.

### *Stone price index with lagged expenditure shares*

Demand elasticities of the LA-AIDS model based on the Stone price index with lagged expenditure shares (11) can be calculated with the formulas derived for the LA-AIDS model based on the Stone price index with current expenditure shares (see Section 4.2). This is because demand elasticities of static models indicate variations between two equilibria and in an equilibrium expenditure shares become steady ( $w_{it} = w_{it-1}$ ) so that the two Stone price indices become equivalent. Therefore, `aidsEla` uses formulas (28) to (33) not only if argument `priceIndex` is "S", but also if it is "SL".

### *Paasche price index*

Formulas for calculating demand elasticities of the LA-AIDS model with Paasche price index (12) can be derived in a similar way as for the model with Stone price index. [Buse's 1994](#) first approach that leads to (28) and (29) for the LA-AIDS with Stone price index yields the following formulas for the LA-AIDS with Paasche price index.

$$\Theta_{ij}^{B1,P} = -\delta_{ij} + \frac{\gamma_{ij}^P}{w_i} - \frac{\beta_i^P}{w_i} \left( \frac{w_j + \sum_k \gamma_{kj}^P \ln \frac{p_k}{p_{k0}}}{1 + \sum_k \beta_k^P \ln \frac{p_k}{p_{k0}}} \right) \quad (34)$$

$$\eta_i^{B1,P} = 1 + \frac{\beta_i^P}{w_i \left( 1 + \sum_k \beta_k^P \ln \frac{p_k}{p_{k0}} \right)} \quad (35)$$

Function `aidsElas` calculates demand elasticities with these formulas, if argument `priceIndex` is "P" and argument `method` is "B1" or "GA".

Formulas for the LA-AIDS with Paasche price index that correspond to (30) and (31) can be obtained in a similar way.

$$\Theta_{ij}^{B2,P} = -\delta_{ij} + \frac{\gamma_{ij}^P}{w_i} - \frac{\beta_i^P}{w_i} \left( w_j + \sum_k \gamma_{kj}^P \ln \frac{p_k}{p_{k0}} - \sum_k \beta_k^P \ln \frac{p_k}{p_{k0}} \left( \alpha_j + \sum_l \gamma_{lj}^P \ln p_l \right) \right) \quad (36)$$

$$\eta_i^{B2,P} = 1 + \frac{\beta_i^P}{w_i} \left( 1 - \sum_k \beta_k^P \ln \frac{p_k}{p_{k0}} \right) \quad (37)$$

These formulas are used by `aidsElas`, if argument `priceIndex` is "P" and argument `method` is "B2".

If argument `priceIndex` is "P" and argument `method` is "Go" or "Ch" `aidsElas` uses the same formulas as for the LA-AIDS with Stone price index, i.e. (32) and (20).

*Laspeyres price indices*

Since the Laspeyres price index (13) does not include current expenditure shares, the approaches of Buse (1994) and Goddard (1983), which yield formulas (28) to (32) for the LA-AIDS model with Stone price index, lead all to the same result for the LA-AIDS model with Laspeyres price index.

$$\Theta_{ij}^{B1/B2/Go,L} = -\delta_{ij} + \frac{\gamma_{ij}^L}{w_i} - \frac{\beta_i^L}{w_i} w_{j0} \quad (38)$$

The formula for calculating expenditure elasticities is identical to the formula for the original AIDS (20). The derivation of demand elasticities for the LA-AIDS model with the simplified Laspeyres price index (15) leads to the same formulas as for the model with the non-simplified version of this price index.

Function `aidsElas` uses these formulas, if its argument `priceIndex` is "L" or "Ls" and its argument `method` is "B1", "GA", "B2", "Go", or "Ch".

*Tornqvist price index*

Formulas for calculating demand elasticities of the LA-AIDS model with Tornqvist price index (14) can be derived in a similar way as for the models with Stone or Paasche price index. Buse's 1994 first approach yields the following formulas.

$$\Theta_{ij}^{B1,T} = -\delta_{ij} + \frac{\gamma_{ij}^T}{w_i} - \frac{\beta_i^T}{2w_i} \left( \frac{w_j + w_{j0} + \sum_k \gamma_{kj}^T \ln \frac{p_k}{p_{k0}}}{1 + \frac{1}{2} \sum_k \beta_k^T \ln \frac{p_k}{p_{k0}}} \right) \quad (39)$$

$$\eta_i^{B1,T} = 1 + \frac{\beta_i^T}{w_i \left( 1 + \frac{1}{2} \sum_k \beta_k^T \ln \frac{p_k}{p_{k0}} \right)} \quad (40)$$

Function `aidsElas` calculates demand elasticities with these formulas, if argument `priceIndex` is "T" and argument `method` is "B1" or "GA".

Formulas for the LA-AIDS with Tornqvist price index that correspond to (30) and (31) can be obtained in a similar way.

$$\Theta_{ij}^{B2,T} = -\delta_{ij} + \frac{\gamma_{ij}^T}{w_i} - \frac{\beta_i^T}{2w_i} \left( w_j + w_{j0} + \sum_k \gamma_{kj}^T \ln \frac{p_k}{p_{k0}} - \sum_k \beta_k^T \ln \frac{p_k}{p_{k0}} \left( \alpha_j + \sum_l \gamma_{lj}^T \ln p_l \right) \right) \quad (41)$$

$$\eta_i^{B2,T} = 1 + \frac{\beta_i^T}{w_i} \left( 1 - \frac{1}{2} \sum_k \beta_k^T \ln \frac{p_k}{p_{k0}} \right) \quad (42)$$

These formulas are used by `aidsElas`, if argument `priceIndex` is "T" and argument `method` is "B2".

Goddard's 1983 approach that leads to (32) for the LA-AIDS with Stone price index yields following formula for the LA-AIDS with Tornqvist price index.

$$\Theta_{ij}^{Go,T} = -\delta_{ij} + \frac{\gamma_{ij}^T}{w_i} - \frac{\beta_i^T}{2w_i} (w_j + w_{j0}) \quad (43)$$

The expenditure elasticities can be obtained with the same formula as for the original AIDS (20). If argument `priceIndex` is "T" and argument `method` is "Go" or "Ch", `aidsElas` uses these formulas for calculating demand elasticities.

## 5. Prediction of expenditure shares and demanded quantities

In empirical demand analysis, it is often desirable to use an estimated demand model for predictions and simulations. In this section it is demonstrated how expenditure shares and demanded quantities can be predicted with the AIDS and LA-AIDS model.

### 5.1. Predictions with the AIDS

If prices, total expenditure, and the coefficients of the AIDS are given, demanded quantities and expenditure shares can easily be calculated with equations (2), (3), and (4). These calculations are implemented in function `aidsCalc`. It is generally called by

```
> aidsCalc( priceNames, totExpName, coef, data )
```

The four mandatory arguments `priceNames`, `totExpName`, `coef`, and `data` are identical to the mandatory arguments of function `aidsMono` (see Section 3.2). We demonstrate the usage of `aidsCalc` by using the AIDS model estimated in Section 2.4 to calculate the expected expenditure shares and quantities in the (hypothetical) case that the meat prices would have been 10% higher than the actual prices.<sup>5</sup> First we create a new data frame `B86new` with meat prices 10% higher than in the original data set.

```
> B86new <- Blanciforti86
> B86new$pFood1 <- B86new$pFood1 * 1.10
```

Now we use `aidsCalc` to calculate the demanded quantities and expenditure shares. Argument `coef` can be set to the object returned by the `coef` method for `aidsEst`, because it has the same structure as required by this argument.

```
> aidsMeat10 <- aidsCalc( priceNames, "xFood",
+   coef( aidsResult ), data = B86new )
```

Function `aidsCalc` returns a list with two elements: `shares` is a data frame that contains the calculated expenditure shares and `quant` is a data frame that contains the calculated quantities. A comparison of the simulated quantities and expenditure shares with the fitted values shows the effect of the hypothetical price increase. To save space, we compare only the quantities of the last year of the sample.

```
> aidsMeat10$quant[ 32, ] / fitted( aidsResult )$quant[ 32, ]
```

```
      q1    q2    q3    q4
1978 0.907 0.928 1.01 1.04
```

A 10% price increase for meat would decrease the demand for meat (`q1`) by 9% and the demand for fruits and vegetables (`q2`) by 7%, while it would increase the demand for cereals and bakery products (`q3`) by 1% and demand for miscellaneous foods (`q4`) by 4%.

---

<sup>5</sup>For simplicity we assume that the increase of the meat prices has no effect on total food expenditure (`xFood`).



## 5.2. Predictions with the LA-AIDS

While the calculation of demanded quantities and expenditure shares is straightforward for the AIDS, it is more complicated for the LA-AIDS with Stone price index, because all current expenditure shares appear also on the right-hand side of each demand equation. However, this system of equations can be solved for the expenditure shares by matrix algebra.

$$w_t = (I + \beta \ln p_t')^{-1} (\alpha + \gamma \ln p_t + \beta \ln m_t) \quad (44)$$

where  $w_t = (w_{1t}, w_{2t}, \dots, w_{Nt})'$ ,  $p_t = (p_{1t}, p_{2t}, \dots, p_{Nt})'$ ,  $\alpha$  is the vector of all  $\alpha_i$ s,  $\beta$  is the vector of all  $\beta_i$ s,  $\gamma$  is the matrix of all  $\gamma_{ij}$ s,  $I$  is an  $N \times N$  identity matrix and  $N$  is the number of goods. Function `aidsEst` uses this formula, if its argument `priceIndex` is "S".

```
> laaidsMeat10 <- aidsCalc( priceNames, "xFood",
+   coef( laaidsResult ), data = B86new, priceIndex = "S" )
```

The calculations of demanded quantities and expenditure shares from the LA-AIDS with Stone price index using lagged shares require (fitted) expenditure shares in the period before each observation for calculating the price index for this observation. Since (fitted) expenditure shares in the period before the first observation are usually unknown, these calculations are generally not feasible. As a workaround, `aidsCalc` calculates the expenditure shares for the first observation with (44). Then all following expenditure shares can be calculated consecutively by (11) and (9). Function `aidsEst` does these calculations, if its argument `priceIndex` is set to "SL".

```
> laaidsSLMeat10 <- aidsCalc( priceNames, "xFood",
+   coef( laaidsResultSL ), data = B86new, priceIndex = "SL" )
```

For the LA-AIDS with Laspeyres or simplified Laspeyres price index, the demanded quantities and expenditure shares can be easily calculated with (9), (13), and (15). The function `aidsEst` does these calculations if its argument `priceIndex` is set to "L" or "Ls", respectively. Since the estimated coefficients are valid only for a specific vector of base prices and base expenditure shares, the same values of  $p_{k0}$  and  $w_{k0}$  that have been used to calculate the price index for the estimation have to be used for the calculation of the demanded quantities and expenditure shares; the base prices and base expenditure shares can be specified with arguments `basePrices` and `baseShares`, respectively.

```
> laaidsLMeat10 <- aidsCalc( priceNames, "xFood",
+   coef( laaidsResultL ), data = B86new, priceIndex = "L",
+   basePrices = laaidsResultL$basePrices,
+   baseShares = laaidsResultL$baseShares )
> laaidsLsMeat10 <- aidsCalc( priceNames, "xFood",
+   coef( laaidsResultLs ), data = B86new, priceIndex = "Ls",
+   baseShares = laaidsResultLs$baseShares )
> all.equal( laaidsLMeat10$quant, laaidsLsMeat10$quant, check.attributes = FALSE )
```

[1] TRUE

As noted in Section 2.3, these two models are equivalent and hence, lead to the same simulation results.

The LA-AIDS with Paasche or Tornqvist index has the same problem as the LA-AIDS with Stone index: current expenditure shares on the right-hand side of each demand equation. The expenditure shares can be calculated similarly to (44).

$$\text{Paasche index: } w_t = \left( I + \beta \ln \frac{p'_t}{p'_0} \right)^{-1} (\alpha + \gamma \ln p_t + \beta \ln m_t) \quad (45)$$

$$\text{Tornqvist Index: } w_t = \left( I + \frac{1}{2} \beta \ln \frac{p'_t}{p'_0} \right)^{-1} \left( \alpha + \gamma \ln p_t + \beta \ln m_t - \frac{1}{2} \beta \frac{p'_t}{p'_0} w_0 \right) \quad (46)$$

As expected, `aidsEst` can do these calculations, if its argument `priceIndex` is set to "P" or "T", respectively, and if the required base prices and base expenditure shares are provided.

```
> laaidsPMeat10 <- aidsCalc( priceNames, "xFood",
+   coef( laaidsResultP ), data = B86new, priceIndex = "P",
+   basePrices = laaidsResultP$basePrices )
> laaidsTMeat10 <- aidsCalc( priceNames, "xFood",
+   coef( laaidsResultT ), data = B86new, priceIndex = "T",
+   basePrices = laaidsResultT$basePrices,
+   baseShares = laaidsResultT$baseShares )
```

### 5.3. The predict method

The commands for calculating predicted values can be simplified by using the `predict` method for objects of class `aidsEst`. It extracts all relevant elements from an object returned by `aidsEst` and passes them as arguments to `aidsCalc`.

```
> predict( object, newdata = NULL, observedShares = FALSE )
```

The optional argument `newdata` can be used to provide a data frame which contains the variables for the prediction. If it is omitted, the data frame used for the estimation is used also for prediction so that fitted values are returned. The optional argument `observedShares` determines whether fitted or observed expenditure shares are used in the calculation of the Stone, Paasche, and Tornqvist price index. It is `FALSE` by default, which means that fitted expenditure shares are used. Observed expenditure shares are used, if this argument is `TRUE`.

Now we demonstrate the usage of the `predict` method for objects of class `aidsEst` by a few examples and show that it returns the same values as the calls to `aidsCalc` above.

```
> aidsMeat10a <- predict( aidsResult, newdata = B86new )
> all.equal( aidsMeat10, aidsMeat10a )
```

```
[1] TRUE
```

```
> laaidsMeat10a <- predict( laaidsResult, newdata = B86new )
> all.equal( laaidsMeat10, laaidsMeat10a )
```

[1] TRUE

## 6. Replication exercises

In this section, the **micEconAids** package is used to replicate two demand analyses that are published in the literature. This has several reasons. First, a comparison of **micEconAids**'s results with results published in the literature confirms the reliability of the **micEconAids** package. Second, this section helps researchers, demand analysts, teachers, and students to become familiar with using the **micEconAids** package. Third, the section encourages reproducible research, which should be a general goal in scientific analysis (Buckheit and Donoho 1995; Schwab, Karrenbach, and Claerbout 2000). For instance, by preparing this section, some undocumented data transformations and the exact estimation methods in the analysis of Blanciforti *et al.* (1986) have been discovered.

### 6.1. Blanciforti, Green, and King (1986)

First, we reproduce a demand analysis of Blanciforti *et al.* (1986), which is also the source of our data set `Blanciforti86`. They apply the linear approximation of the AIDS and substitute the Stone price index with lagged shares (11) for the translog price index. Among other analyses, they examine the demand for the four food groups with a static LA-AIDS model. Blanciforti *et al.* (1986) do not use their data as published in their appendix, but scale prices and total expenditure to one in the year 1972. To make our result comparable to theirs, we do the same.

```
> priceNamesScaled <- paste( priceNames, "Scaled", sep = " " )
> for( i in 1:4 ){
+   Blanciforti86[[ priceNamesScaled[ i ] ]] <-
+     Blanciforti86[[ priceNames[i] ]] /
+     Blanciforti86[ "1972", priceNames[ i ] ]
+ }
> Blanciforti86$xFoodScaled <- Blanciforti86$xFood /
+   Blanciforti86[ "1972", "xFood" ]
```

As Blanciforti *et al.* (1986) estimate their model by “Full Information Maximum Likelihood” (FIML), which is not yet available in **systemfit** (Henningsen and Hamann 2007a,b), we apply the “Iterated Seemingly Unrelated Regression” (ITSUR) estimator, which generally converges to FIML results (Greene 2003, p. 347). This can be done by adding argument `maxiter` to `aidsEst`, which will be passed to **systemfit**. If `maxiter` is larger than one, the SUR estimation will be iterated until the coefficients converge or the number of iterations is `maxiter`.

First, we estimate the model with homogeneity and symmetry imposed. The results of Blanciforti *et al.* (1986) are presented part “A” of table 5.1 (p. 36).

```
> estResultLA <- aidsEst( priceNamesScaled, shareNames, "xFoodScaled",
+   data = Blanciforti86, priceIndex = "SL", maxiter = 100 )
> estResultLA
```

Demand analysis with the Almost Ideal Demand System (AIDS)  
 Estimation Method: Linear Approximation (LA) with lagged Stone Index (SL)

Coefficients:

alpha

wFood1	wFood2	wFood3	wFood4
0.327	0.209	0.129	0.336

beta

wFood1	wFood2	wFood3	wFood4
0.3252	0.0522	-0.0750	-0.3024

gamma

	pFood1Scaled	pFood2Scaled	pFood3Scaled	pFood4Scaled
wFood1	0.108	-0.1396	-0.010950	0.042969
wFood2	-0.140	0.1605	-0.004704	-0.016219
wFood3	-0.011	-0.0047	0.016032	-0.000377
wFood4	0.043	-0.0162	-0.000377	-0.026373

```
> elas( estResultLA )$exp
```

q_wFood1	q_wFood2	q_wFood3	q_wFood4
2.039	1.257	0.438	0.136

```
> elas( estResultLA )$marshall
```

	pFood1Scaled	pFood2Scaled	pFood3Scaled	pFood4Scaled
q_wFood1	-0.9816	-0.657	-0.1736	-0.226
q_wFood2	-0.7668	-0.263	-0.0574	-0.170
q_wFood3	0.0938	0.079	-0.8049	0.194
q_wFood4	0.3932	0.129	0.1142	-0.773

All coefficients and elasticities are close to the published values. The only exception in the Marshallian own price elasticity of the third food group (cereal and bakery products). However, this seems to be a typo in [Blanciforti et al. \(1986\)](#), because the same elasticities are presented in table 5.3 again and this table shows a value that is close to our result. The small differences between our and the published results are probably due to different estimation methods (FIML vs. ITSUR) and rounding errors in the data as only 3 digits of the budget shares and prices are presented in the appendix of [Blanciforti et al. \(1986\)](#).

Now, we estimate the model without homogeneity and symmetry imposed. Apparently, [Blanciforti et al. \(1986\)](#) use scaled prices, but non-scaled total food expenditure in this estimation. Their results are presented in the columns labelled with “NH” in table 5.2 (p. 39).

```
> estResultLANh <- aidsEst( priceNamesScaled, shareNames, "xFood",
+   data = Blanciforti86, priceIndex = "SL", hom = FALSE, sym = FALSE,
+   maxiter = 100 )
> estResultLANh
```

Demand analysis with the Almost Ideal Demand System (AIDS)

Estimation Method: Linear Approximation (LA) with lagged Stone Index (SL)

Coefficients:

alpha

wFood1	wFood2	wFood3	wFood4
-0.567	0.255	0.538	0.773

beta

wFood1	wFood2	wFood3	wFood4
0.13999	-0.00762	-0.06431	-0.06806

gamma

	pFood1Scaled	pFood2Scaled	pFood3Scaled	pFood4Scaled
wFood1	0.11952	-0.0422	-0.05554	0.01002
wFood2	-0.12691	0.1547	0.02755	-0.04370
wFood3	-0.00528	-0.0255	0.03142	-0.00107
wFood4	0.01268	-0.0870	-0.00344	0.03475

> estResultLANh\$r2

wFood1	wFood2	wFood3	wFood4
0.816	0.864	0.438	0.857

> elas( estResultLANh )\$exp

q_wFood1	q_wFood2	q_wFood3	q_wFood4
1.448	0.963	0.518	0.806

> elas( estResultLANh )\$marshall

	pFood1Scaled	pFood2Scaled	pFood3Scaled	pFood4Scaled
q_wFood1	-0.7576	-0.2262	-0.2375	-0.125
q_wFood2	-0.6130	-0.2311	0.1406	-0.202
q_wFood3	0.1110	-0.0928	-0.7003	0.161
q_wFood4	0.0968	-0.2085	0.0161	-0.833

While most estimated coefficients,  $R^2$  values, and expenditure elasticities are very close to the published values, some Marshallian elasticities considerably differ. The reason for these differences has not been discovered yet.

As a further example, we estimate the model with homogeneity but without symmetry imposed. Again, [Blanciforti et al. \(1986\)](#) use scaled prices but non-scaled total food expenditure. Their results are presented in the columns labelled with “NS” in table 5.3 (p. 41).

```
> estResultLAns <- aidsEst( priceNamesScaled, shareNames, "xFood",
+   data = Blanciforti86, priceIndex = "SL", sym = FALSE, maxiter = 100 )
> estResultLAns
```

Demand analysis with the Almost Ideal Demand System (AIDS)

Estimation Method: Linear Approximation (LA) with lagged Stone Index (SL)

Coefficients:

```

alpha
wFood1 wFood2 wFood3 wFood4
-1.764 -0.182  0.553  2.393
beta
  wFood1  wFood2  wFood3  wFood4
  0.3282  0.0611 -0.0666 -0.3227
gamma
      pFood1Scaled pFood2Scaled pFood3Scaled pFood4Scaled
wFood1      0.10607      -0.1183      -0.0479      0.06010
wFood2     -0.13182       0.1269       0.0304     -0.02542
wFood3     -0.00512      -0.0245       0.0313     -0.00167
wFood4      0.03087       0.0160      -0.0138     -0.03302

> estResultLAns$r2

wFood1 wFood2 wFood3 wFood4
  0.689  0.850  0.438  0.715

> elas( estResultLAns )$exp

q_wFood1 q_wFood2 q_wFood3 q_wFood4
  2.0481  1.3003  0.5013  0.0781

> elas( estResultLAns )$marshall

      pFood1Scaled pFood2Scaled pFood3Scaled pFood4Scaled
q_wFood1     -0.989     -0.5910     -0.2928     -0.175
q_wFood2     -0.742     -0.4371      0.1092     -0.230
q_wFood3      0.118     -0.0824     -0.6988      0.162
q_wFood4      0.377      0.2331      0.0836     -0.772

```

Also these results are mostly very close to the values published in [Blanciforti et al. \(1986\)](#).

In addition to the LA-AIDS, [Blanciforti et al. \(1986\)](#) estimate an original non-linear AIDS. As the lagged shares in the Stone price index of the LA-AIDS require to drop the first observation in the LA-AIDS, this observation is also dropped in the estimation of the AIDS. We estimate this AIDS model by “Iterative Linear Least Squares” (ILLE). The results of [Blanciforti et al. \(1986\)](#) are presented in the column labelled with “AIDS” and “p = 0” in table 5.4 (p. 42).

```

> estResultA <- aidsEst( priceNames, shareNames, "xFood",
+   data = Blanciforti86[ -1, ], method = "IL", maxiter = 100 )
> elas( estResultA )$exp

q_wFood1 q_wFood2 q_wFood3 q_wFood4
  2.046  1.293  0.458  0.102

> diag( elas( estResultA )$marshall )

```

```
[1] -0.995 -0.279 -0.808 -0.772
```

The estimated elasticities are rather close to the published values, but not as close as the values in the previous estimations. This is probably due to different non-linear optimization routines in TSP (used by [Blanciforti \*et al.\* 1986](#)) and `aidsEst`.

## 6.2. Green and Alston (1990)

[Green and Alston \(1990\)](#) compare Marshallian own price elasticities of the AIDS with elasticities of the LA-AIDS model that are obtained by different formulas (see section 4.2). While the elasticities of the AIDS model are taken from [Blanciforti \*et al.\* \(1986, table 5.4, p. 42\)](#), the elasticities of the LA-AIDS are calculated from the coefficients of an own LA-AIDS estimation.

```
> diag( elas( estResultA, method = "AIDS" )$marshall )
```

```
[1] -0.995 -0.279 -0.808 -0.772
```

```
> estResultLA <- aidsEst( priceNames, shareNames, "xFood",
+   data = Blanciforti86, priceIndex = "S", maxiter = 100 )
> diag( elas( estResultLA, method = "AIDS" )$marshall )
```

```
[1] -0.414 -0.238 -0.734 -0.318
```

```
> diag( elas( estResultLA, method = "EU" )$marshall )
```

```
[1] -0.667 -0.209 -0.888 -1.067
```

```
> diag( elas( estResultLA, method = "Ch" )$marshall )
```

```
[1] -0.991 -0.264 -0.811 -0.765
```

```
> diag( elas( estResultLA, method = "GA" )$marshall )
```

```
[1] -0.999 -0.264 -0.809 -0.761
```

Most estimated elasticities are extremely close to the published values. Only some elasticities from the non-linear AIDS model slightly deviate. As said before, these deviations probably come from different non-linear optimization routines in TSP (used by [Blanciforti \*et al.\* 1986](#)) and `aidsEst`.

## 7. Summary and Conclusions

The R package `micEconAids` provides a convenient interface for demand analyses with the Almost Ideal Demand System (AIDS) and its linear approximation (LA-AIDS). The functions `aidsEst` and `aidsEla` can estimate coefficients and demand elasticities of the LA-AIDS using various different approaches that solve the severe problems originating from the linear

approximation of the non-linear AIDS. However, some of the problems of the LA-AIDS seem to be generally intractable. Therefore, the question arises whether the linear approximation should be applied at all. Since the estimation of the non-linear AIDS often leads to problems in empirical applications, `aidsEst` applies the “Iterative Linear Least-Squares Estimator” (Blundell and Robin 1999) for the econometric estimation of the non-linear AIDS. A special feature of `aidsEla` is the calculation of the variance-covariance matrix and the standard errors of the estimated demand elasticities. These allow the calculation of confidence intervals for the elasticities and the application of statistical tests. The reliability of `aidsEst` and `aidsEla` is demonstrated by replicating two demand analyses that are published in the literature.

Since the publication of the AIDS, several generalizations and extensions of this model have been proposed, e.g., the “Generalized AIDS” (GAIDS, Bollino 1987), the “Quadratic AIDS” (QAIDS, Banks, Blundell, and Lewbel 1997), and a version that accounts for censoring of the dependent variables (Shonkwiler and Yen 1999). We plan to implement these enhancements in future versions of the **micEconAids** package — possibly with the help of users that would like to apply these extended models.

## Acknowledgments

The author thanks an anonymous referee and several users of the **micEconAids** package for their comments and suggestions. Of course, any remaining errors are the author’s.

## References

- Alston JM, Foster KA, Green RD (1994). “Estimating Elasticities with the Linear Approximate Almost Ideal Demand System: Some Monte Carlo Results.” *The Review of Economics and Statistics*, **76**(2), 351–356.
- Anderson GJ, Blundell RW (1983). “Testing Restrictions in a Flexible Dynamic Demand System: An Application to Consumers’ Expenditure in Canada.” *Review of Economic Studies*, **50**(3), 397–410.
- Asche F, Wessells CR (1997). “On Price Indices in the Almost Ideal Demand System.” *American Journal of Agricultural Economics*, pp. 1182–1185.
- Banks J, Blundell RW, Lewbel A (1997). “Quadratic Engel Curves and Consumer Demand.” *The Review of Economics and Statistics*, **79**(4), 527–539.
- Barten AP (1969). “Maximum Likelihood Estimation of a Complete System of Demand Equations.” *European Economic Review*, **1**(1), 7–73.
- Blanciforti LA, Green R (1983). “An Almost Ideal Demand System Incorporating Habits: An Analysis of Expenditures on Food and Aggregate Commodity Groups.” *The Review of Economics and Statistics*, **65**(3), 511–515.
- Blanciforti LA, Green RD, King GA (1986). “U.S. Consumer Behavior Over the Postwar Period: An Almost Ideal Demand System Analysis.” Monograph Number 40 (August 1986), Giannini Foundation of Agricultural Economics, University of California.



- Blundell RW, Robin JM (1999). "Estimation in Large and Disaggregated Demand Systems: An Estimator for Conditionally Linear Systems." *Journal of Applied Econometrics*, **14**(3), 209–232.
- Bollino CA (1987). "GAIDS: A Generalized Version of the Almost Ideal Demand System." *Economics Letters*, **23**(2), 199–202.
- Brännlund R, Ghalwash T, Nordström J (2007). "Increased Energy Efficiency and the Rebound Effect: Effects on Consumption and Emissions." *Energy Economics*, **29**(1), 1–17.
- Browning M, Meghir C (1991). "The Effects of Male and Female Labor Supply on Commodity Demands." *Econometrica*, **59**, 925–951.
- Buckheit J, Donoho DL (1995). "Wavelab and Reproducible Research." In A Antoniadis (ed.), *Wavelets and Statistics*. Springer-Verlag.
- Buse A (1993). "Evaluating the Linearized Almost Ideal Demand System." *Research Paper 93-18*, Department of Economics, University of Alberta, Edmonton.
- Buse A (1994). "Evaluating the Linearized Almost Ideal Demand System." *American Journal of Agricultural Economics*, **76**(4), 781–793.
- Buse A (1998). "Testing Homogeneity in the Linearized Almost Ideal Demand System." *American Journal of Agricultural Economics*, **80**, 208–220.
- Buse A, Chan WH (2000). "Invariance, Price Indices and Estimation in Almost Ideal Demand Systems." *Empirical Economics*, **25**, 519–539.
- Chalfant JA (1987). "A Globally Flexible, Almost Ideal Demand System." *Journal of Business and Economic Statistics*, **5**, 233–242.
- Chambwera M, Folmer H (2007). "Fuel Switching in Harare: An Almost Ideal Demand System Approach." *Energy Policy*, **35**(4), 2538–2548.
- Deaton A, Muellbauer J (1980a). "An Almost Ideal Demand System." *The American Economic Review*, **70**(3), 312–326.
- Deaton A, Muellbauer J (1980b). *Economics and Consumer Behaviour*. Cambridge University Press, Cambridge.
- Eales JS, Unnevehr LJ (1988). "Demand for Beef and Chicken Products: Separability and Structural Change." *American Journal of Agricultural Economics*, **70**, 521–532.
- Farrell L, Shields MA (2007). "Children as Consumers: Investigating Child Diary Expenditure Data." *Canadian Journal of Economics*, **40**(2), 445–467.
- Goddard D (1983). "An Analysis of Canadian Aggregate Demand for Food at Home and Away from Home." *Canadian Journal of Agricultural Economics*, **31**, 289–318.
- Green R, Alston JM (1990). "Elasticities in AIDS models." *American Journal of Agricultural Economics*, **72**, 442–445.
- Greene WH (2003). *Econometric Analysis*. 5th edition. Prentice Hall.

- Hahn WF (1994). “Elasticities in AIDS Models: Comment.” *American Journal of Agricultural Economics*, **76**, 972–977.
- Hausman JA, Leonard GK (2007). “Estimation of Patent Licensing Value Using a Flexible Demand Specification.” *Journal of Econometrics*, **139**(2), 242–258.
- Henning CHCA, Henningsen A (2007). “Modeling Farm Households’ Price Responses in the Presence of Transaction Costs and Heterogeneity in Labor Markets.” *American Journal of Agricultural Economics*, **89**(3), 665–681.
- Henningsen A, Hamann JD (2007a). “systemfit: A Package for Estimating Systems of Simultaneous Equations in R.” *Journal of Statistical Software*, **23**(4), 1–40. URL <http://www.jstatsoft.org/v23/i04/>.
- Henningsen A, Hamann JD (2007b). *systemfit: Simultaneous Equation Estimation*. R package version 1.0, <http://CRAN.R-project.org>.
- Huang MH, Jones E, Hahn DE (2007). “Determinants of Price Elasticities for Private Labels and National Brands of Cheese.” *Applied Economics*, **39**(5), 553–563.
- Klein LR (1953). *A Textbook of Econometrics*. Row, Peterson and Co, New York.
- Michalek J, Keyzer MA (1992). “Estimation of a Two-stage LES-AIDS Consumer Demand System for Eight EC Countries.” *European Review of Agricultural Economics*, **19**(2), 137–163.
- Moore T, Green CJ (2007). “A Portfolio Approach to Firms’ Financing Decisions: Evidence from India Using the Almost Ideal Demand System.” In V Murinde (ed.), *Accounting, Banking and Corporate Financial Management in Emerging Economies*, number 7 in Research in Accounting in Emerging Economies, pp. 347–368. Elsevier, Amsterdam.
- Moschini G (1995). “Units of Measurement and the Stone Index in Demand System Estimation.” *American Journal of Agricultural Economics*, **77**, 63–68.
- Pashardes P (1993). “Bias in Estimating the Almost Ideal Demand System with the Stone Index Approximation.” *The Economic Journal*, **103**, 908–915.
- Poi BP (2002). “From the Help Desk: Demand System Estimation.” *Stata Journal*, **2**(4), 403–410.
- R Development Core Team (2008). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL <http://www.R-project.org>.
- Raknerud A, Skjerpen T, Swensen AR (2007). “A Linear Demand System within a Seemingly Unrelated Time Series Equations Framework.” *Empirical Economics*, **32**(1), 105–124.
- SAS (2008a). “Calculating Elasticities in an Almost Ideal Demand System.” <http://support.sas.com/rnd/app/examples/ets/elasticity/index.htm> [accessed 2008-01-27].
- SAS (2008b). “Estimating an Almost Ideal Demand System Model.” <http://support.sas.com/rnd/app/examples/ets/aids/index.htm> [accessed 2008-01-27].

- Schwab M, Karrenbach M, Claerbout J (2000). "Making Scientific Computations Reproducible." *Computing in Science & Engineering*, **2**(6), 61–67.
- Shonkwiler JS, Yen ST (1999). "Two-Step Estimation of a Censored System of Equations." *American Journal of Agricultural Economics*, **81**(4), 972–982.
- West SE, Williams Robertson C I (2007). "Optimal Taxation and Cross-Price Effects on Labor Supply: Estimates of the Optimal Gas Tax." *Journal of Public Economics*, **91**(3/4), 593–617.
- Xiao N, Zarnikau J, Damien P (2007). "Testing Functional Forms in Energy Modeling: An Application of the Bayesian Approach to U.S. Electricity Demand." *Energy Economics*, **29**(2), 158–166.
- Zellner A (1962). "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias." *Journal of the American Statistical Association*, **57**, 348–368.

## A. Explanation of economic terms

This section explains some economic terms that have been used in this paper. They are sorted in order of appearance.

**The expenditure function** indicates the minimum amount of money that a consumer needs to purchase goods from which (s)he can derive a given level of utility while all prices are given.

**Marshallian (uncompensated) demand functions** indicate the “optimal” quantities demanded, given prices and total expenditure, i.e. they indicate the quantities that the consumer can afford and that derive the highest level of utility.

**Shephard’s Lemma** says that the Hicksian (compensated) demand functions can be derived by taking the partial derivatives of the expenditure function with respect to the consumer prices.

**The indirect utility function** indicates the maximal level of utility that a consumer can derive from purchased goods given an amount of money and given all prices.

**Expenditure (income) elasticities** measure the relationship between a change in the demanded quantity and a change in total expenditure (income). If total expenditure (income) increases by 1%, the expenditure (income) elasticity indicates by how many percent the demanded quantity of a specific good changes. In traditional simple microeconomic demand models, it is assumed that the income is entirely spent for consumption. Hence, income and total expenditure are equivalent.

**Marshallian (uncompensated) price elasticities** measure the relationship between a change in the demanded quantity and a change in a consumer price holding total expenditure constant. If a consumer price increases by 1%, the price elasticity indicates by how many percent the demanded quantity of a specific good changes.

**Hicksian (compensated) price elasticities** measure the relationship between a change in the demanded quantity and a change in a consumer price holding the utility level constant. If a consumer price increases by 1%, the price elasticity indicates by how many percent the demanded quantity of a specific good changes.

**Hicksian (compensated) demand functions** indicate the “optimal” quantities demanded, given prices and the utility level, i.e. they indicate the cheapest bundle of quantities that has to be consumed to derive at least the given utility level.

### Affiliation:

Arne Henningsen  
 Department of Food and Resource Economics  
 University of Copenhagen  
 Rolighedsvej 25  
 1958 Frederiksberg C, Denmark  
 Telephone: +45 3533 2274  
 E-mail: [arne.henningsen@gmail.com](mailto:arne.henningsen@gmail.com)  
 URL: <http://www.arne-henningsen.name/>